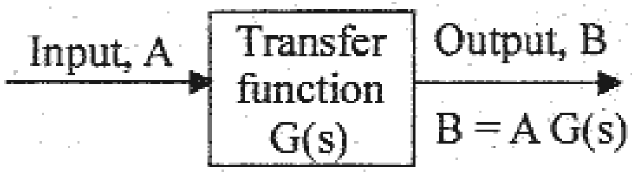
Module-2

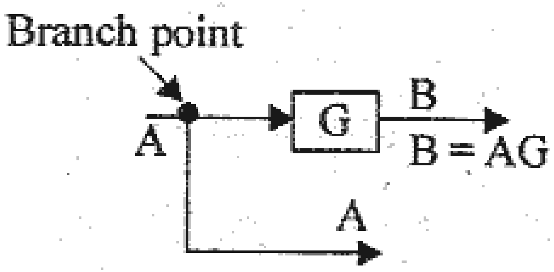
Block diagram: Block diagram of a closed loop system, procedure for drawing block diagram and block diagram reduction to find transfer function. **Signal flow graphs:** Construction of signal flow graphs, basic properties of signal flow graph, signal flow graph algebra, construction of signal flow graph for control systems. (10 Hours)
Revised Bloom's Taxonomy Level: L1 – Remembering, L2 – Understanding, L3 – Applying, L4 – Analysing

BLOCK DIAGRAM

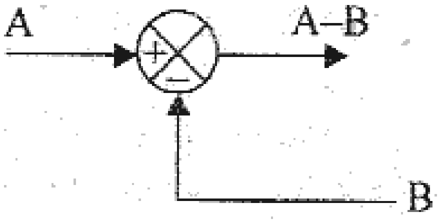
A control system may consist of a number of components. In control engineering to show the functions performed by each component, we commonly use a diagram called the block diagram. A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the interrelationships that exist among the various components.

The elements of a block diagram are block, branch point and summing point.





Branch point



Summing point

Block Diagrams

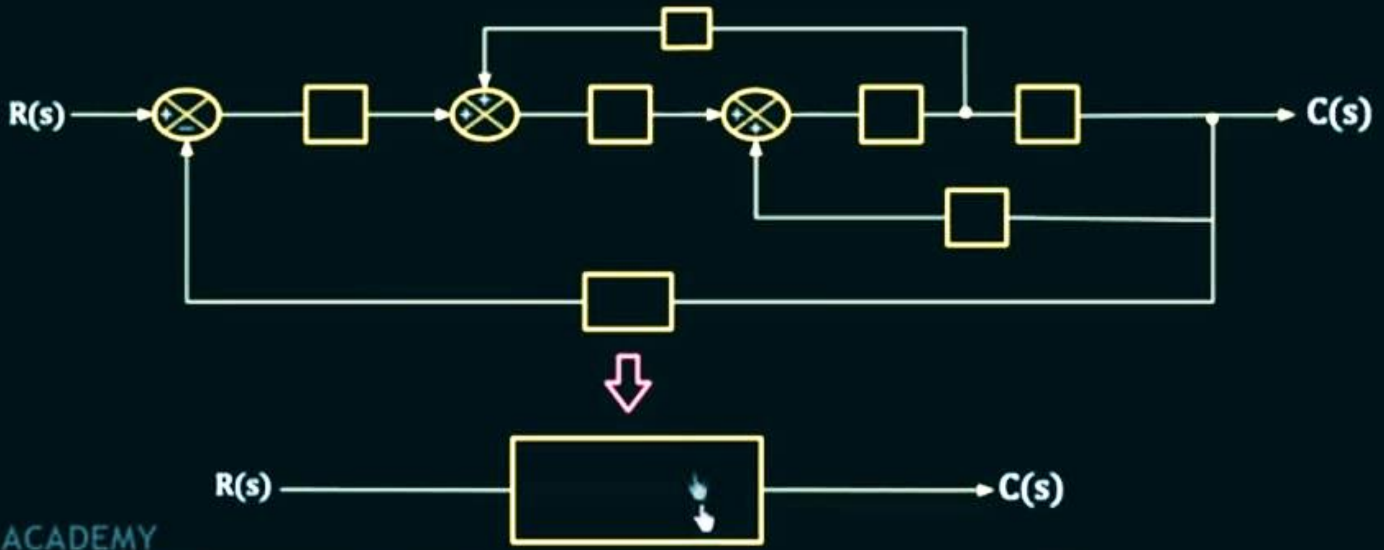
- Block Diagrams are pictorial representation of a control system.
- Block Diagram Representation is used to build a mathematical model of a control system which can be emulated on a computer.
- Block diagram Representation is used to calculate the overall transfer function of the system.

Elements of a Block Diagram:

- The signal into the block represents the input.
- The signal out of the block represents the output.
- The block itself represents the Transfer Function of the system.

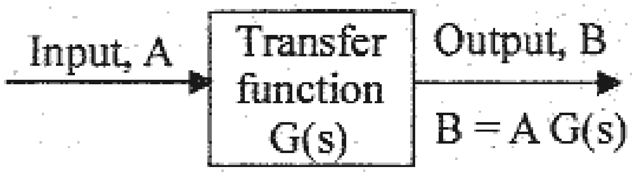
Reduction of Multiple Subsystems

- In order to calculate the output w.r.t any given input, we need the overall transfer function of the system.
- When the system is Complex (having multiple subsystems), we need to reduce the complex block diagram representation into a single block.



BLOCK

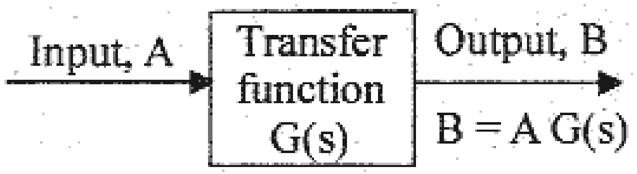
In a block diagram all system variables are linked to each other through functional blocks. The functional block or simply block is a symbol for the mathematical operation on the input signal to the block that produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals. Figure shows the block diagram of functional block.



Functional block

BLOCK

Figure shows the block diagram of functional block.

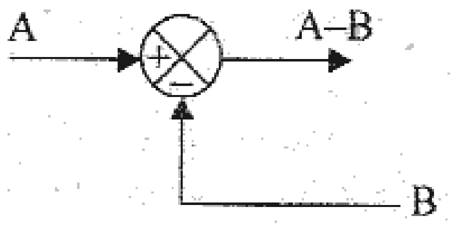


Functional block

The arrowhead pointing towards the block indicates the input, and the arrowhead leading away from the block represents the output. Such arrows are referred to as signals. The output signal from the block is given by the product of input signal and transfer function in the block..

SUMMING POINT

Summing points are used to add two or more signals in the system. Referring to figure, a circle with a cross is the symbol that indicates a summing operation

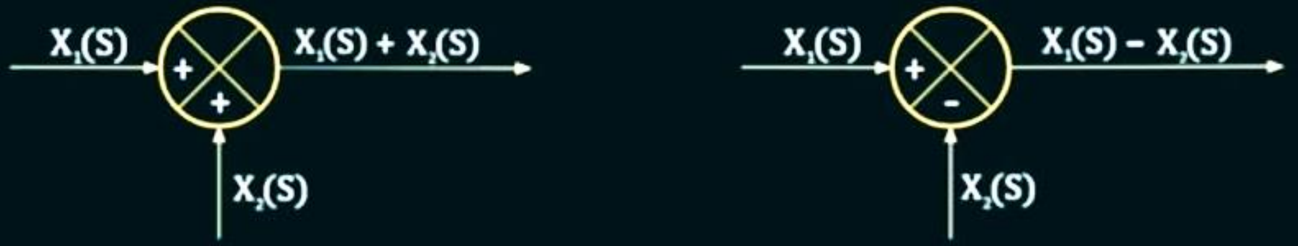


Summing point

The plus or minus sign at each arrowhead indicates whether the signal is to be added or subtracted. It is important that the quantities being added or subtracted have the same dimensions and the same units.

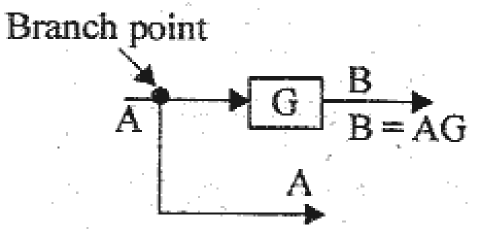
Summing Point:

A Summing Point/Summing Junction in a block diagram represents the dynamic summation of two (or more) signals.



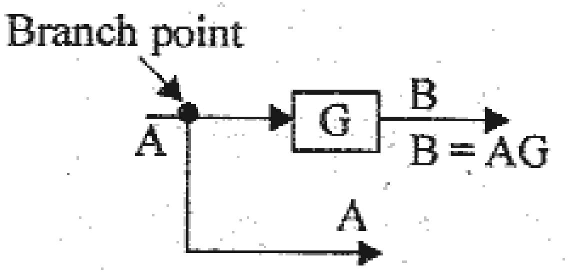
Take-off Point/ Pick-off Point/ Branch point:

Take-off point in a block diagram represents a point where the signal branches out and goes concurrently to the other blocks or summing points.



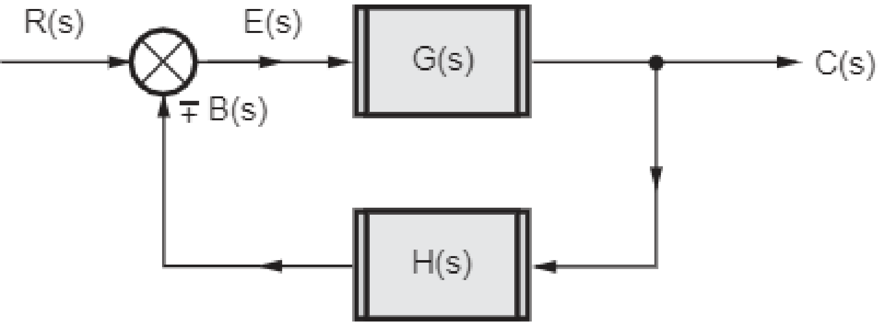
Take-off Point / Branch Point

A branch point is a point from which the signal from a block goes concurrently to other blocks or summing points.



Branch point

Simple or Canonical form of Closed loop System



$R(s) \rightarrow$ Laplace of reference input r(t)

$C(s) \rightarrow$ Laplace of controlled output c(t)

$E(s) \rightarrow$ Laplace of error signal e(t)

$B(s) \rightarrow$ Laplace of feedback signal b(t)

$G(s) \rightarrow$ Equivalent forward path transfer function

$H(s) \rightarrow$ Equivalent feedback path transfer function.

Closed Loop T.F.

$$E(s) = R(s) \mp B(s) \qquad \textbf{1}$$

$$B(s) = C(s)H(s)$$

$$C(s) = E(s)G(s)$$

$$B(s) = C(s)H(s) \text{ and substituting in equation (1)}$$

$$E(s) = R(s) \mp C(s)H(s)$$

$$E(s) = \frac{C(s)}{G(s)}$$

$$\frac{C(s)}{G(s)} = R(s) \mp C(s)H(s)$$

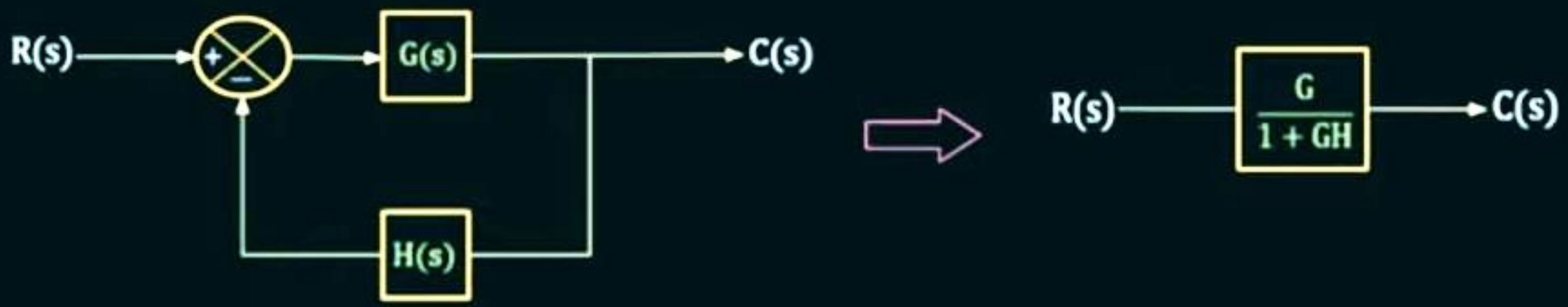
$$C(s) = R(s)G(s) \mp C(s)G(s)H(s)$$

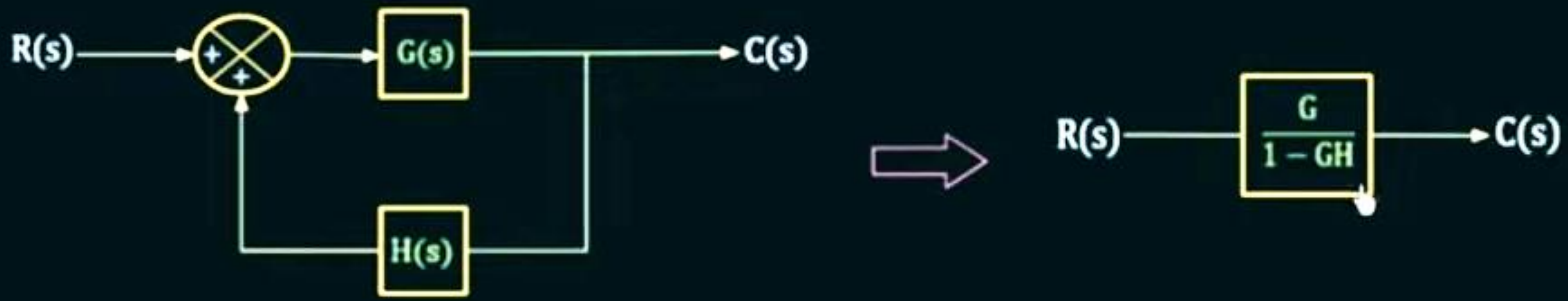
$$\therefore C(s)\,[1 \pm G(s)H(s)] = R(s)\,G(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}}$$

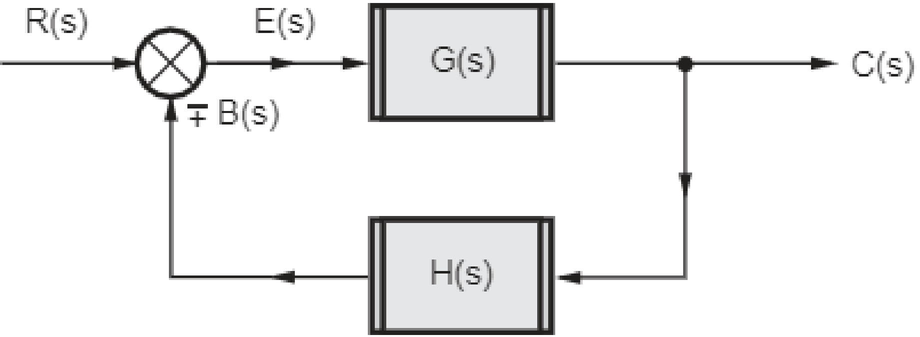
- Use + sign for negative feedback and Use – sign for positive feedback.

Rule 1: Representation of a closed loop system





Simple or Canonical form of Closed loop System



$G(s) \rightarrow$ Equivalent forward path transfer function

$H(s) \rightarrow$ Equivalent feedback path transfer function.

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}}$$

Closed Loop T.F.

- Use + **sign for negative feedback** and Use – **sign for positive feedback.**

CONSTRUCTING BLOCK DIAGRAM FOR CONTROL SYSTEMS

A control system can be represented diagrammatically by block diagram. The differential equations governing the system are used to construct the block diagram. By taking Laplace transform the differential equations are converted to algebraic equations. The equations will have variables and constants. From the working knowledge of the system the input and output variables are identified and the block diagram for each equation can be drawn. Each equation gives one section of block diagram. The output of one section will be input for another section. The various sections are interconnected to obtain the overall block diagram of the system..

BLOCK DIAGRAM REDUCTION

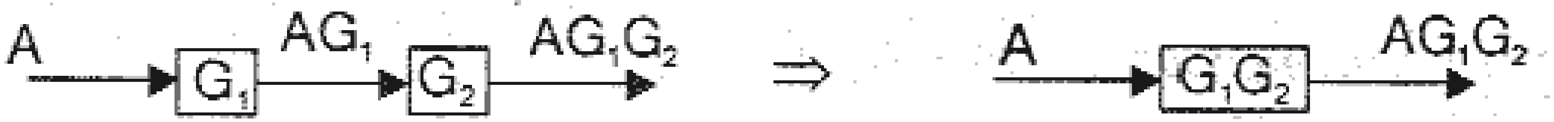
The block diagram can be reduced to find the overall transfer function of the system. The following rules can be used for block diagram reduction. The rules are framed such that any modification made on the diagram does not alter the input-output relation

ATME College of Engineering

Department of EEE
Emitting Elite Energy

RULES OF BLOCK DIAGRAM ALGEBRA

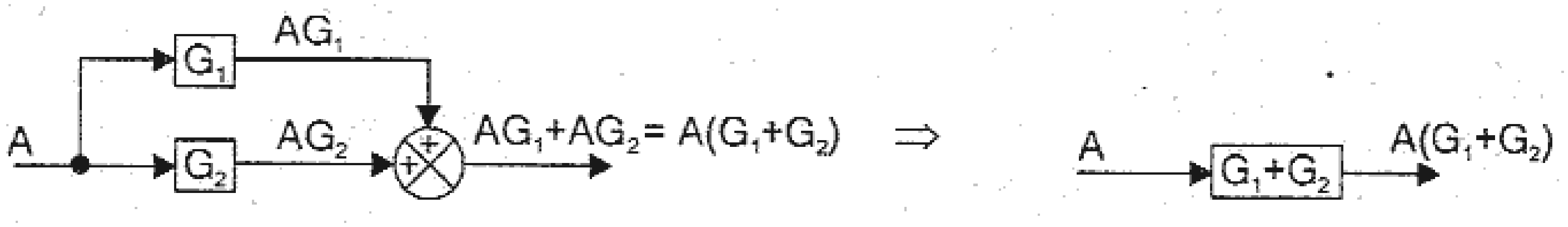
Rule-l: Combining the blocks in cascade



The transfer functions of the blocks which connected in series get multiplied with each other

RULES OF BLOCK DIAGRAM ALGEBRA

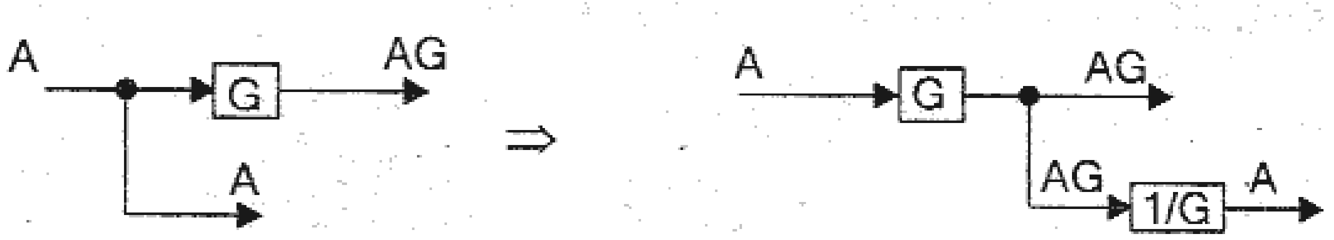
Rule-2: Combining Parallel blocks (or combining feed forward paths)



The transfer functions of the blocks which are connected in parallel get added algebraically (considering the sign)

RULES OF BLOCK DIAGRAM ALGEBRA

Rule-3: Moving/Shifting the Take off Branch Point After a block/Right of the block (Ahead/Beyond)



While shifting a take off point beyond the block, add a block in series with all the signals which are taking off from that point, having T.F. as reciprocal of the T.F. of the block beyond which take off branch is to be shifted.

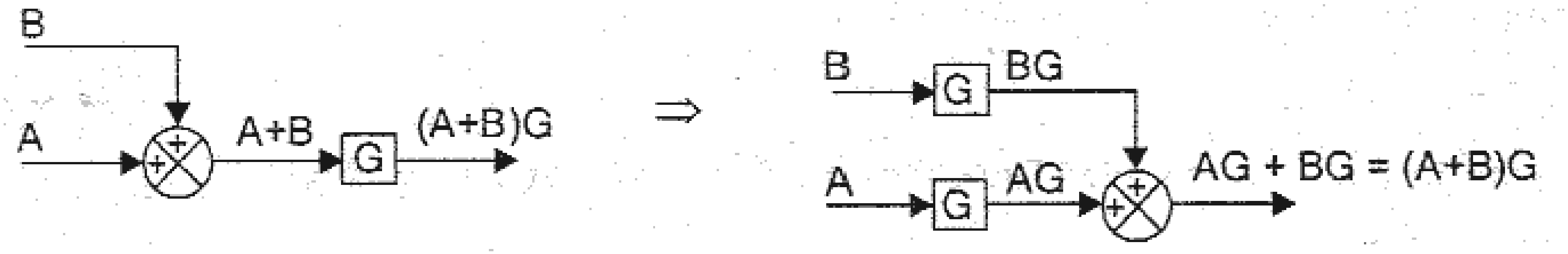
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-4: Moving the branch point before/Behind the block/ Left of the block

While shifting a take off point behind the block, add a block having T.F. same as that of the block behind which take off branch is to be shifted, in series with all the signals taking off from that take off point.

RULES OF BLOCK DIAGRAM ALGEBRA

Rule-5: Moving the summing point after the block i.e Right of the block beyond/ahead/in the front of the block



Thus while shifting a summing point after a block ,add a block having T.F. same as that of block after which summing point is to be shifted, in series with all the signals at that summing point

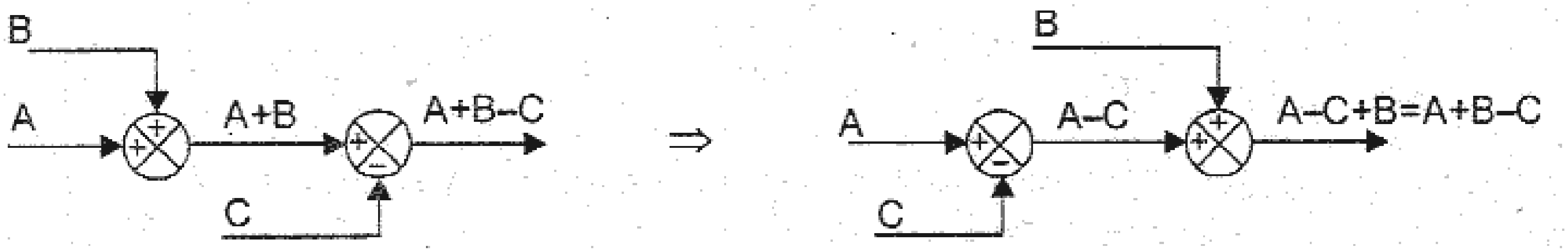
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-6: Moving the summing point before/ Left of the block/Behind the block

Thus while shifting a summing point behind the block i.e. before the block, add a block having T.F. as reciprocal of the T.F. of the block before which summing point is to be shifted, in series with all the signals at that summing point.

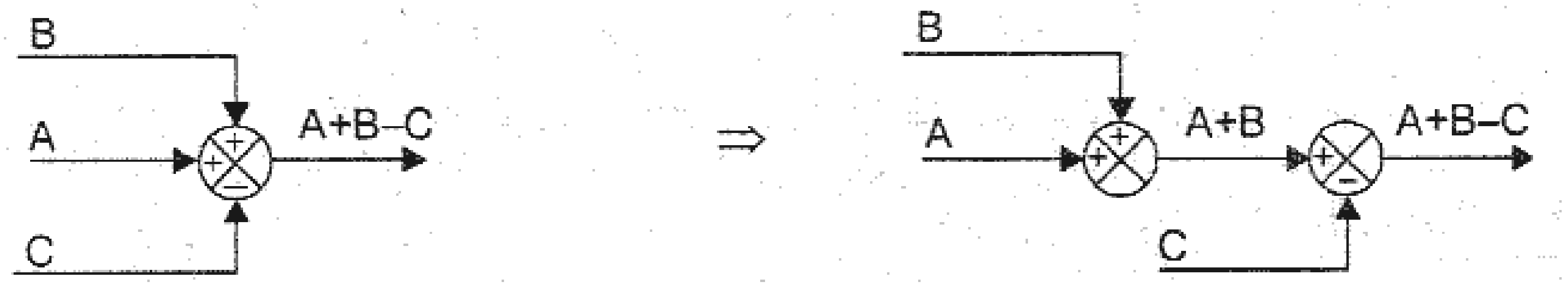
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-7: Interchanging summing point



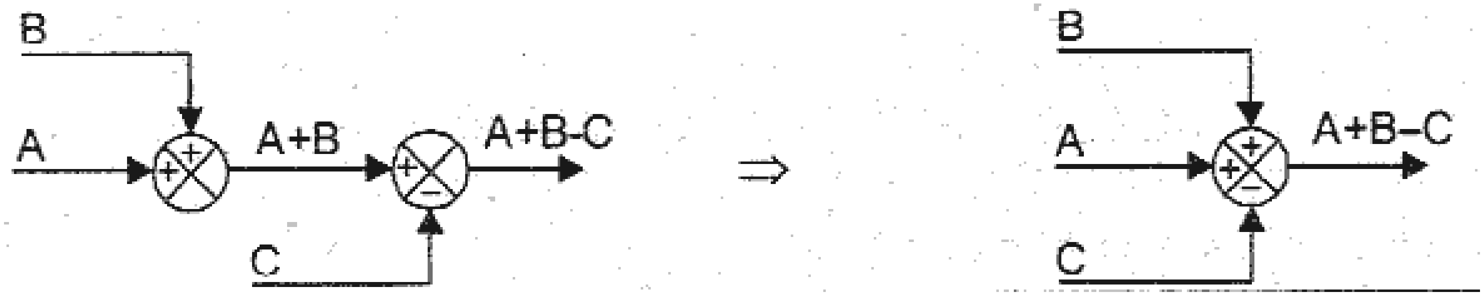
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-8: Splitting summing points



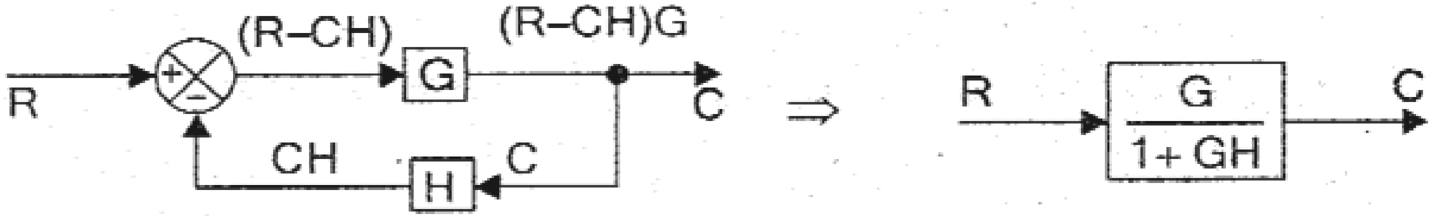
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-9: Combining summing points



RULES OF BLOCK DIAGRAM ALGEBRA

Rule-10: Elimination of (negative) feedback loop



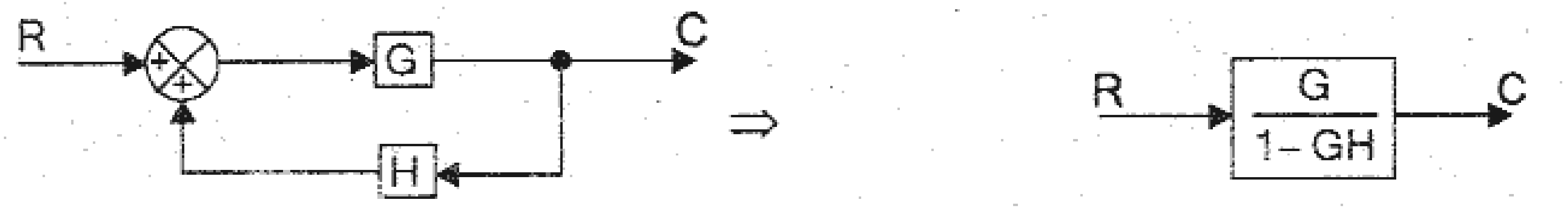
Proof:

$$C = (R - CH)G \quad \Rightarrow \quad C = RG - CHG \quad \Rightarrow \quad C + CHG = RG$$

$$\therefore C(1 + HG) = RG \quad \Rightarrow \quad \frac{C}{R} = \frac{G}{1 + GH}$$

RULES OF BLOCK DIAGRAM ALGEBRA

Rule-11: Elimination of (positive) feedback loop



Procedure to solve BLOCK DIAGRAM Reduction Problem

- Step 1: Reduce the block connected in series
- Step 2: Reduce the block connected in parallel
- Step 3: Reduce the Minor internal Feedback loops
- Step 4: Try to shift branch point(Take off points) towards right and summing point towards left
- Step 5: Repeat step 1 to 4 till simple form is obtained
- Step 6: using standard function of simple closed loop system, obtain the closed loop transfer function C(s)/R(s) of overall system

1.Reduce the block diagram shown in fig 1 and find C/R.

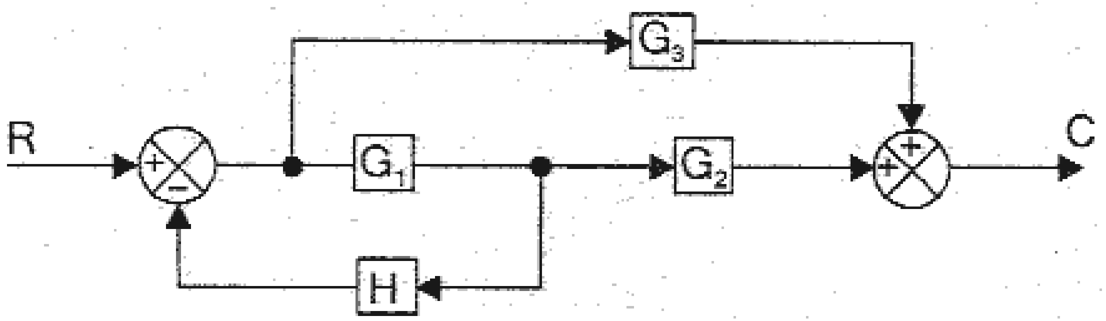


fig 1

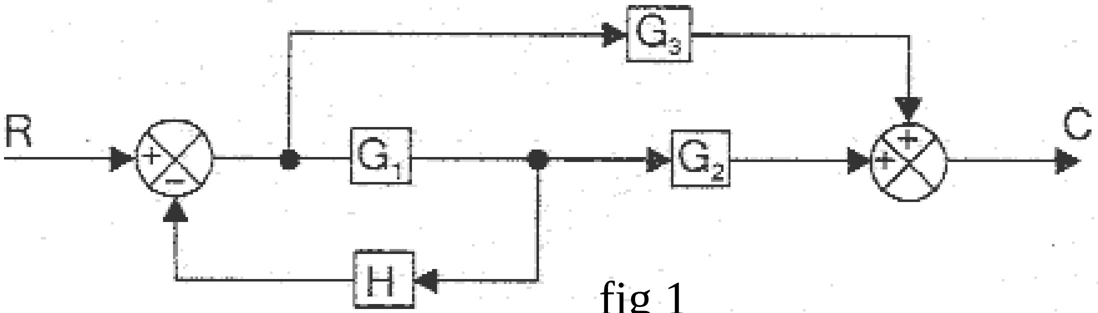
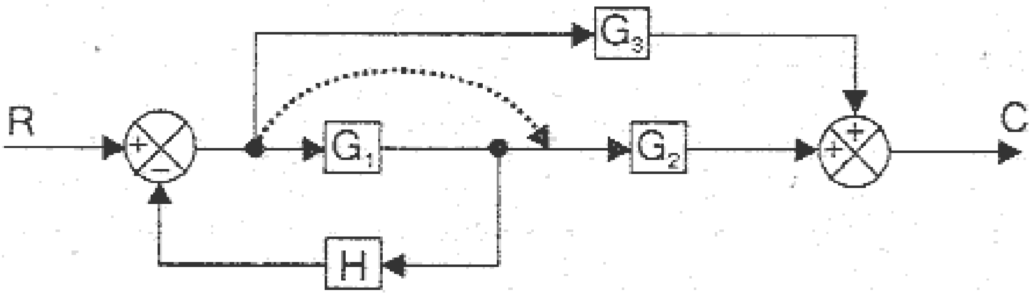
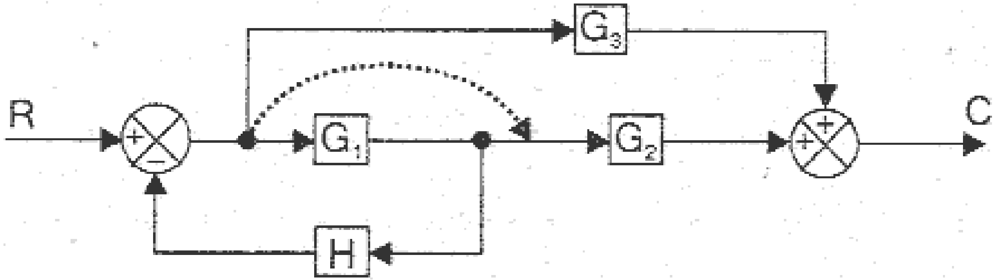


fig 1

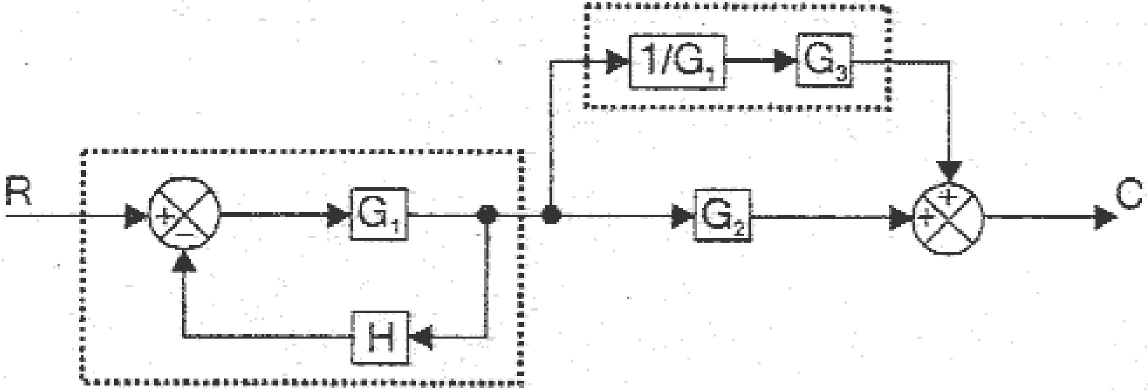
Step 1: Move the branch point after the block.



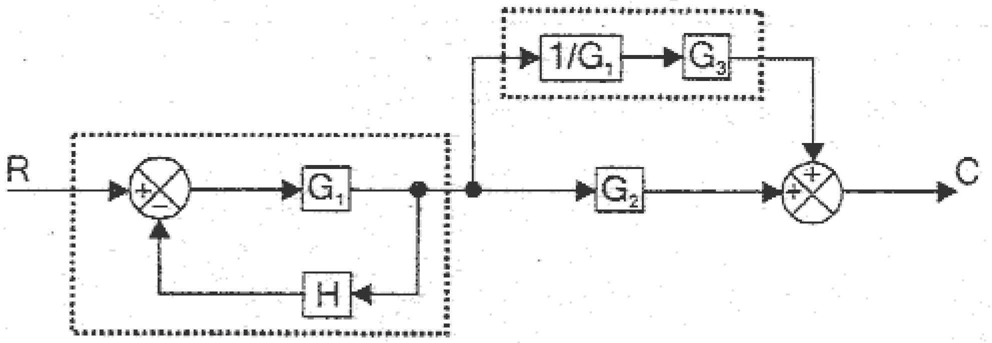
Step 1: Move the branch point after the block.



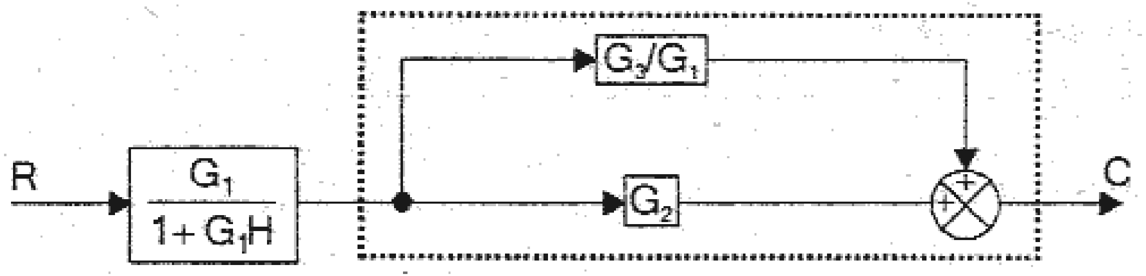
Step 2: Eliminate the feedback path and combining blocks in cascade.



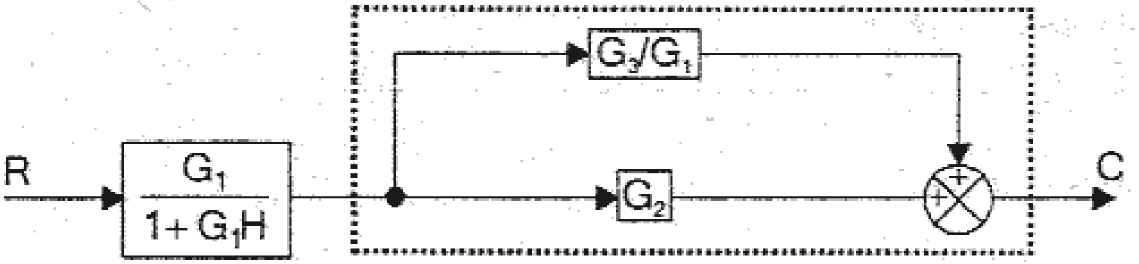
Step 2: Eliminate the feedback path and combining blocks in cascade.



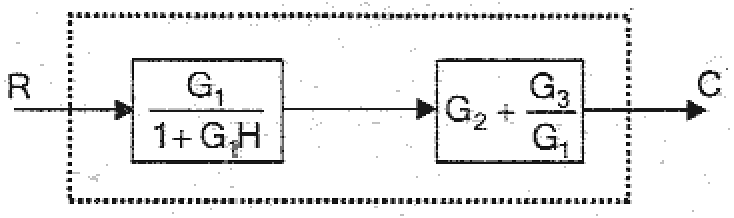
Step 3: Combining parallel blocks



Step 3: Combining parallel blocks

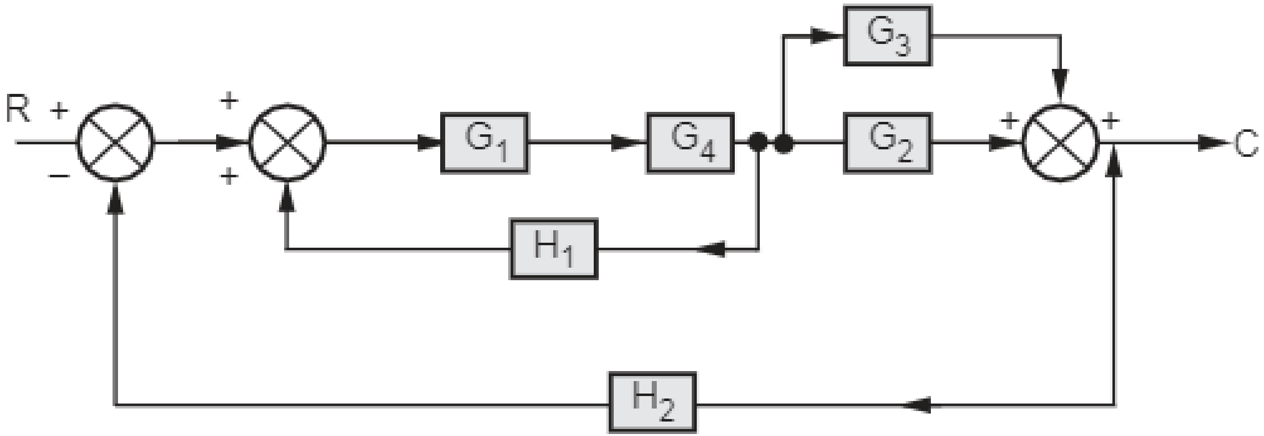


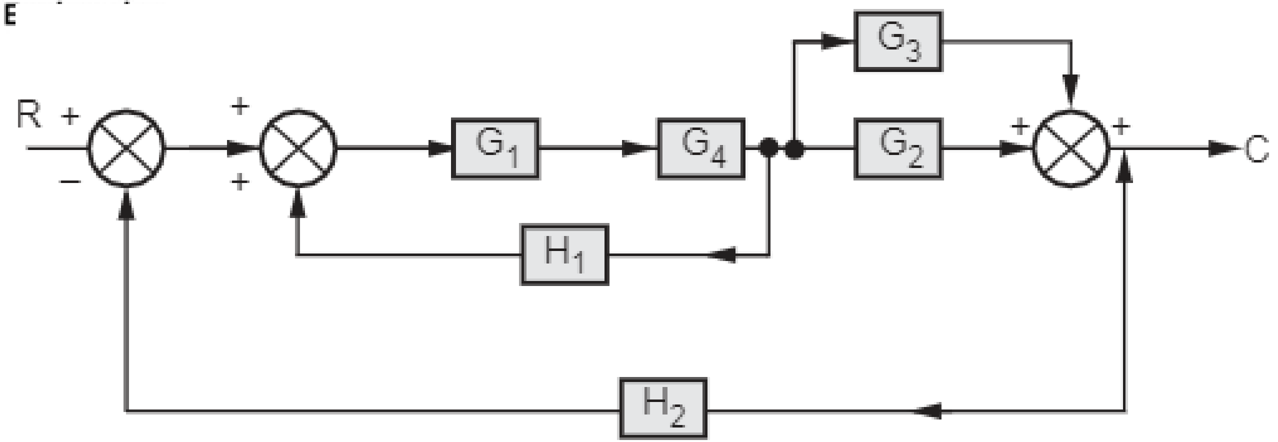
Step 4: Combining blocks in cascade



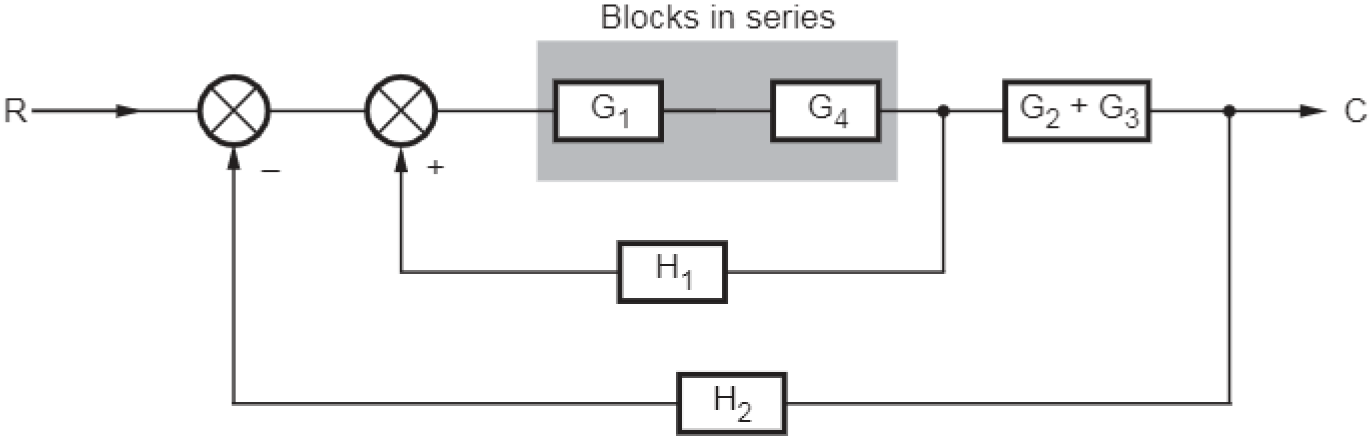
$$\frac{C}{R}=\left(\frac{G_1}{1+G_1H}\right)\left(G_2+\frac{G_3}{G_1}\right)=\left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{G_1}\right)=\frac{G_1G_2+G_3}{1+G_1H}$$

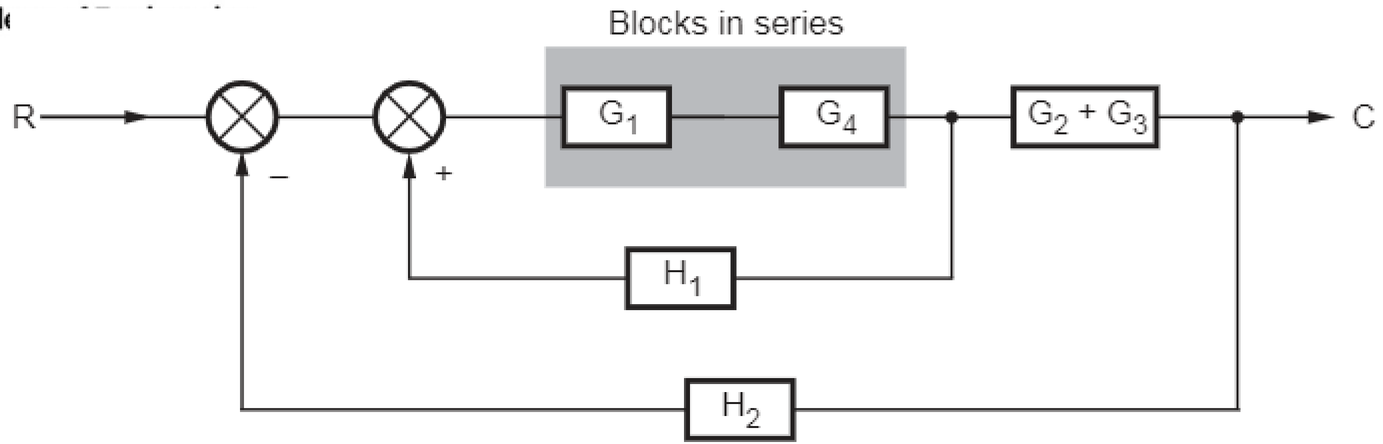
2.Reduce the block diagram shown in figure and find T.F C(S)/R(S).



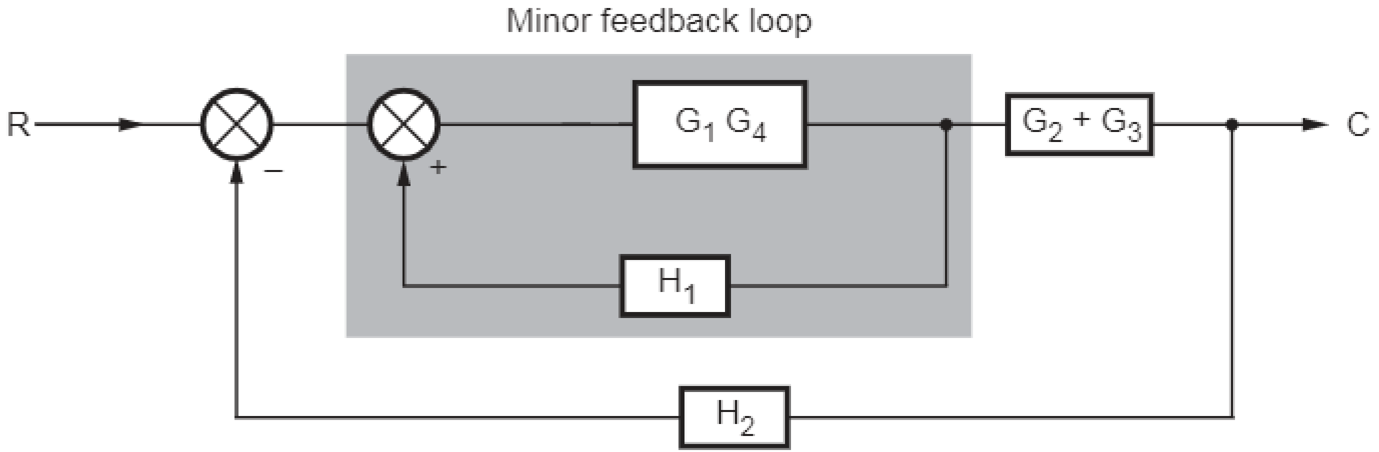


Combine the Parallel block G3 and G2, Combining blocks in cascade (G1 and G4)

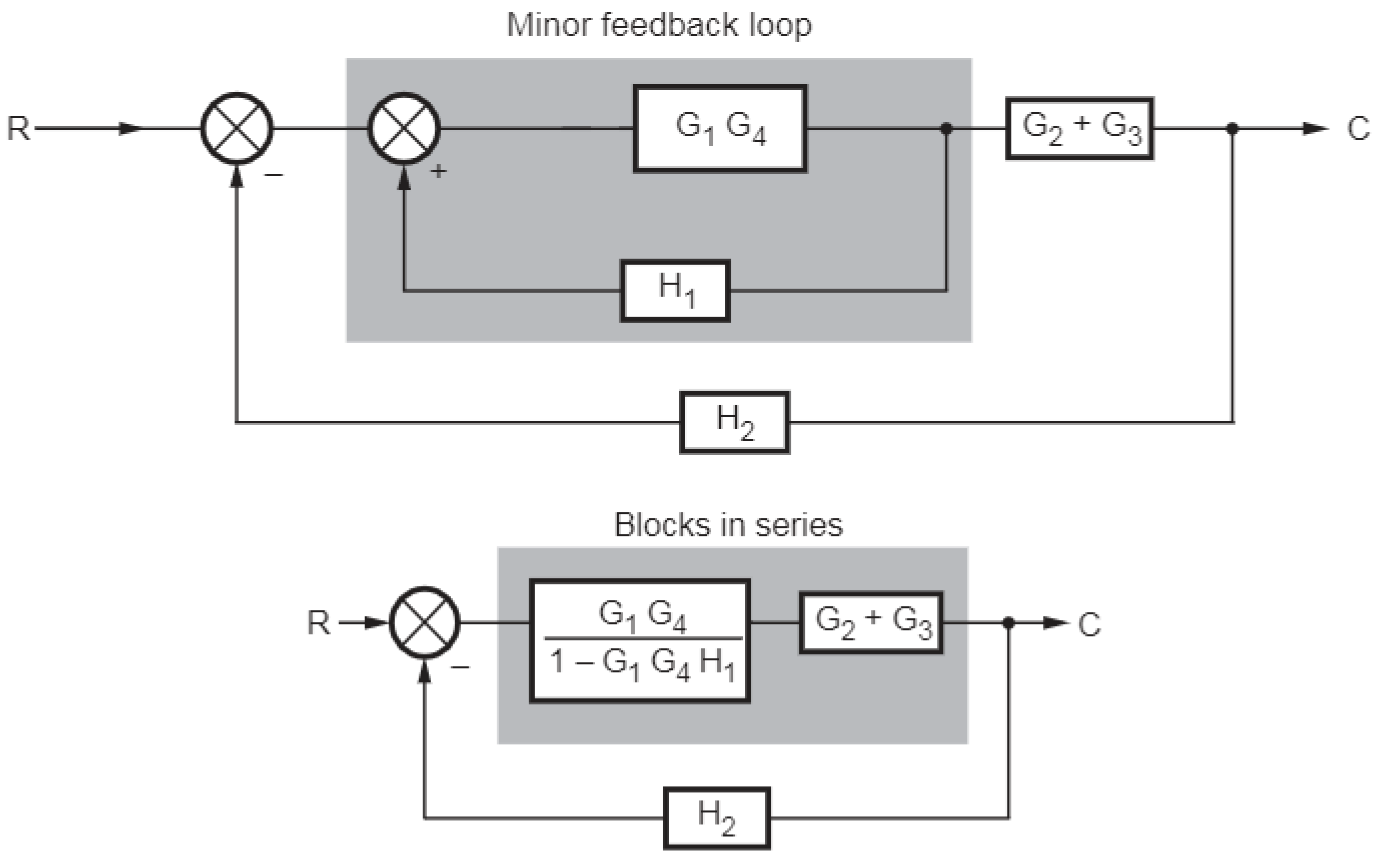




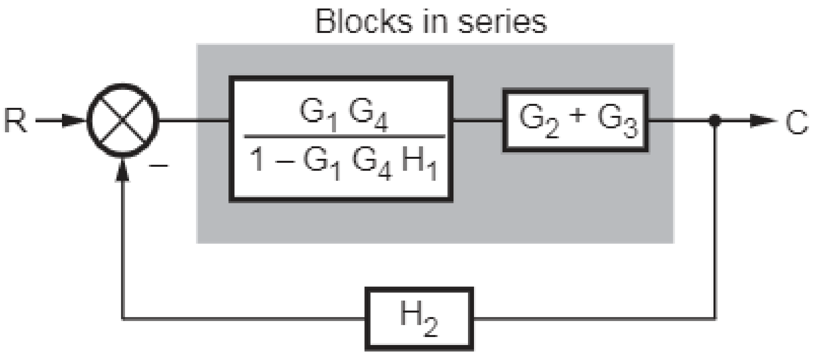
Eliminating feedback path

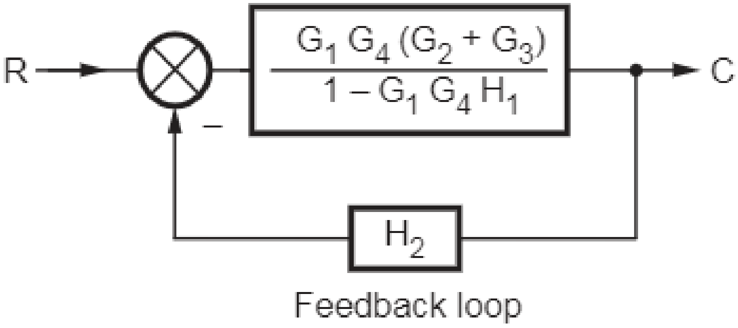


Eliminating feedback path



Combining blocks in cascade and Eliminate feedback path



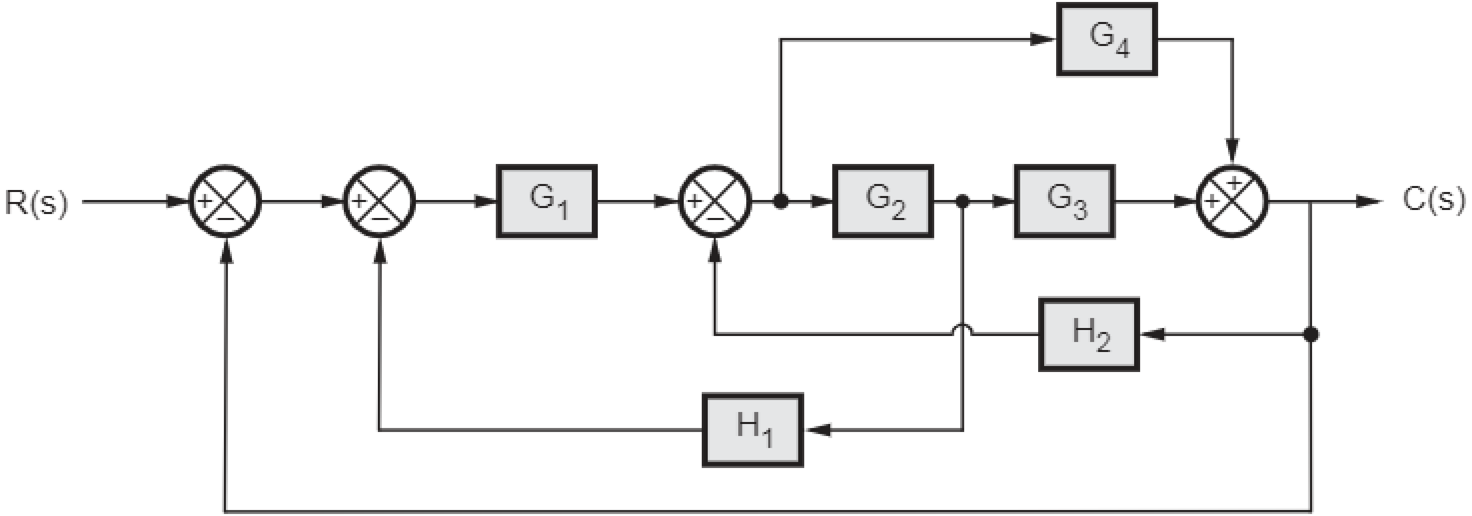


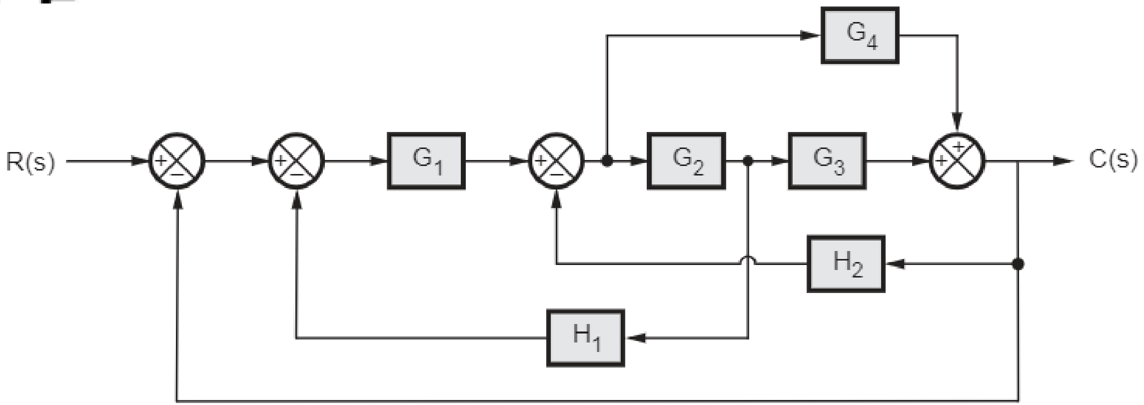
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_4 (G_2 + G_3)}{1 - G_1 G_4 H_1}}{1 + \dfrac{G_1 G_4 (G_2 + G_3) H_2}{1 - G_1 G_4 H_1}} = \frac{G_1 G_4 (G_2 + G_3)}{1 - G_1 G_4 H_1 + G_1 G_4 (G_2 + G_3) H_2}$$

ATME College of Engineering

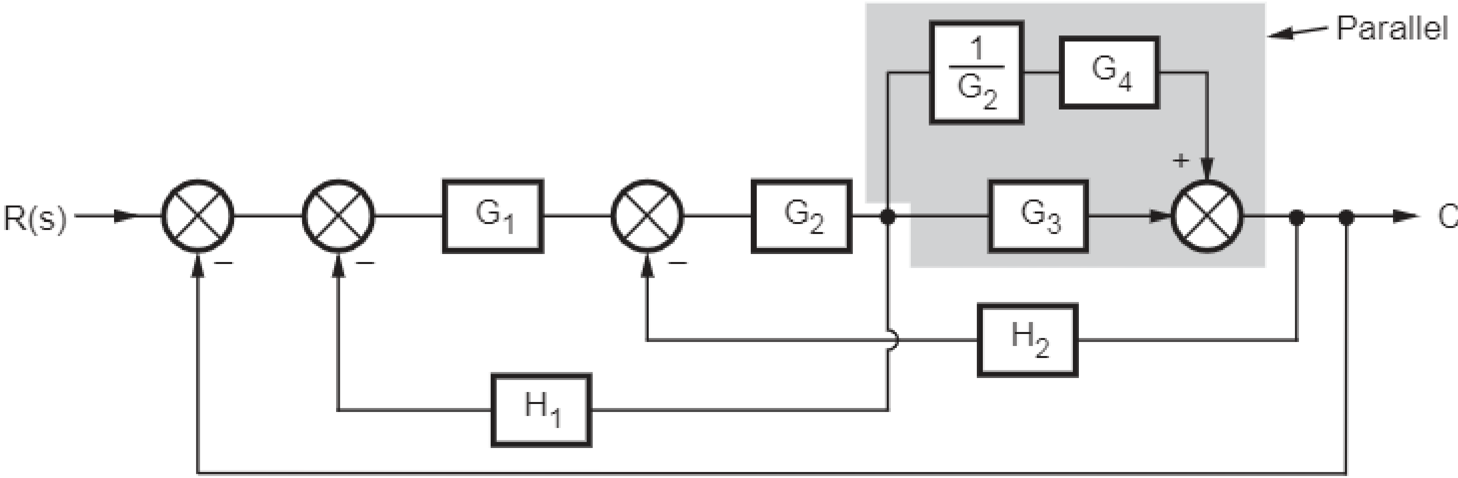
Department of EEE
Emitting Elite Energy

3.Reduce the block diagram shown in figure and find T.F C(S)/R(S).



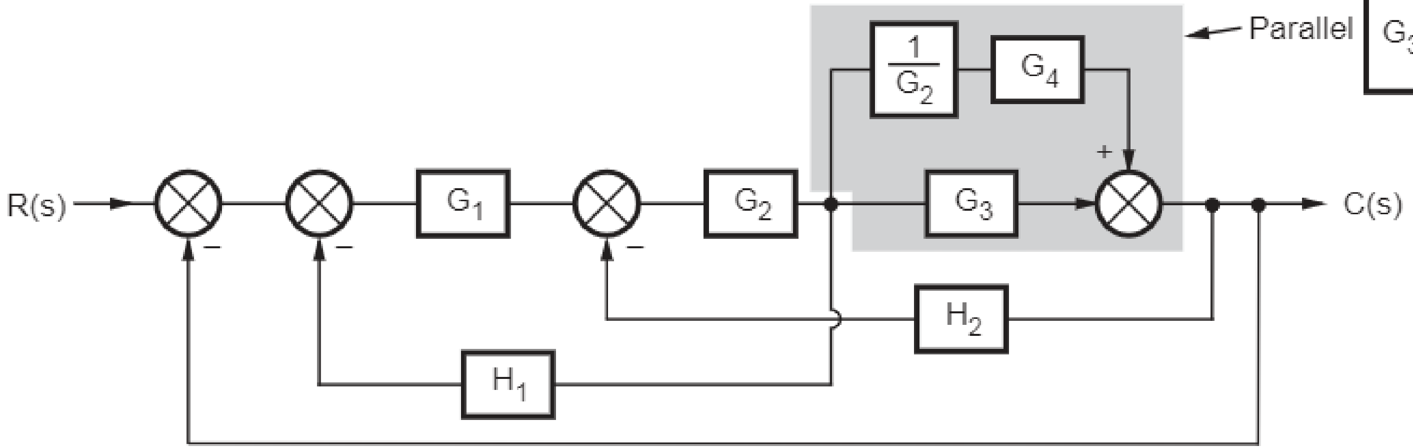


Shift the Branch point/take-off point of G4 after G2 and separate the feedback paths

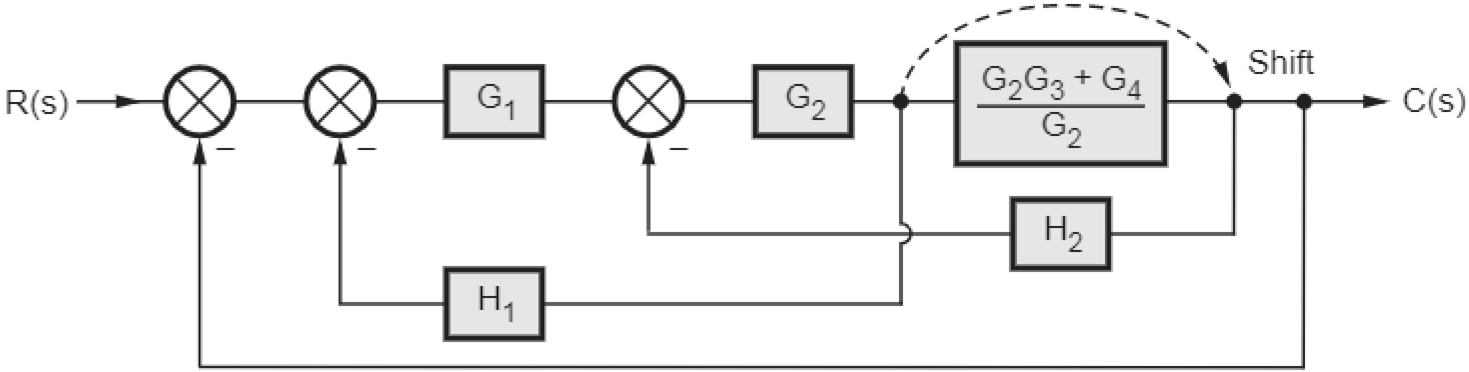


$$G_3 + \frac{G_4}{G_2} = \frac{G_2G_3 + G_4}{G_2}$$

Combine the blocks in cascade and eliminating parallel blocks



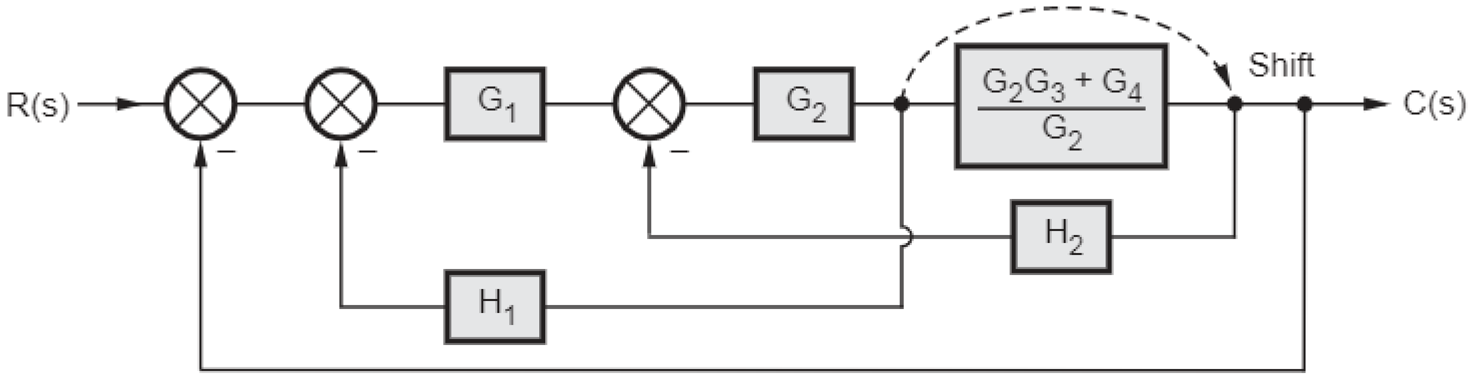
$$G_3 + \frac{G_4}{G_2} = \frac{G_2G_3 + G_4}{G_2}$$

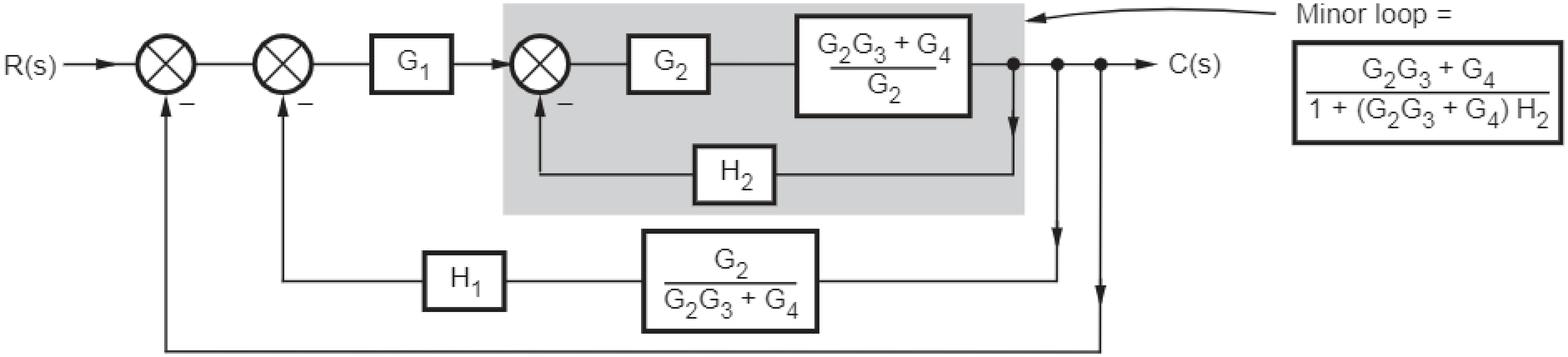


ATME College of Engineering

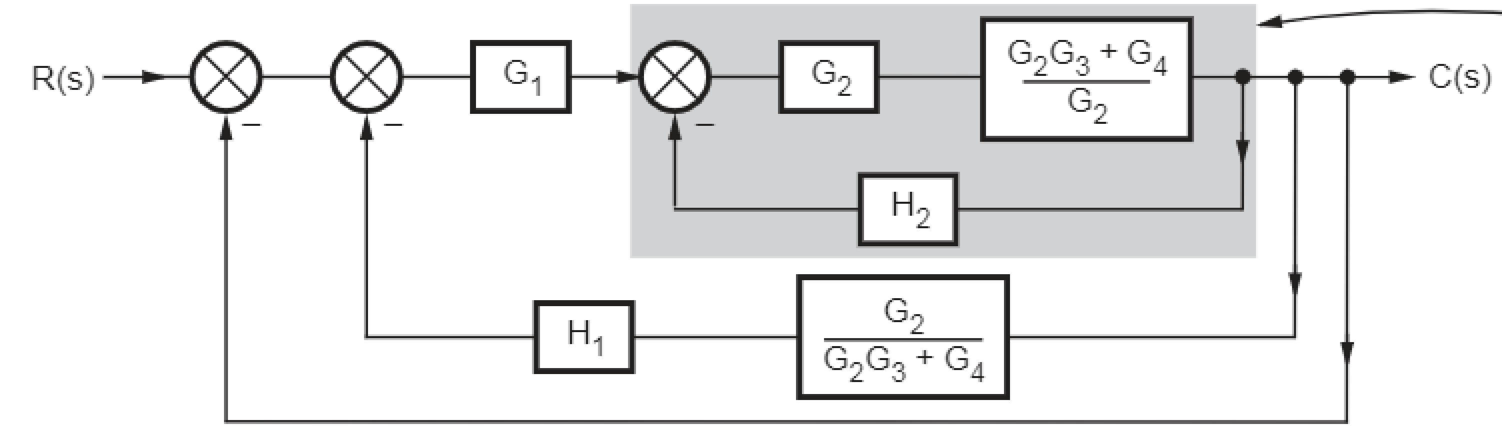
Department of EEE
Emitting Elite Energy

Moving the branch point/Take off point after the block



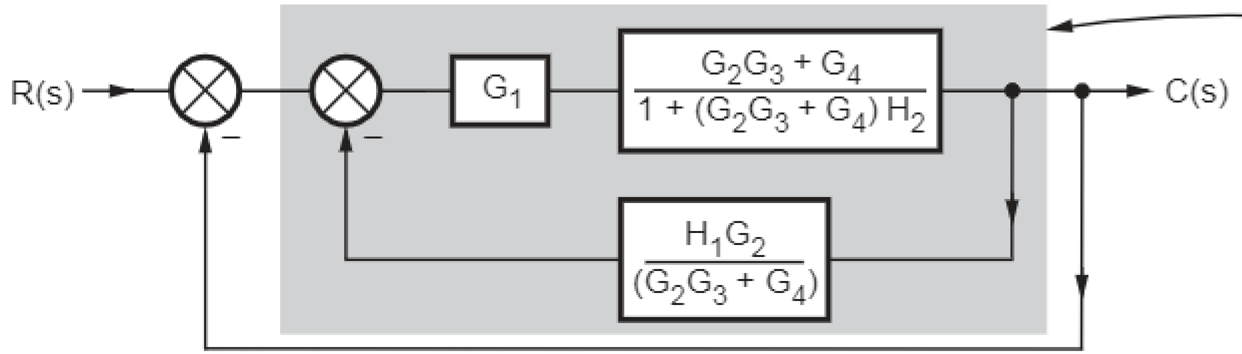


Combine the blocks in cascade and eliminating feedback path



Minor loop =

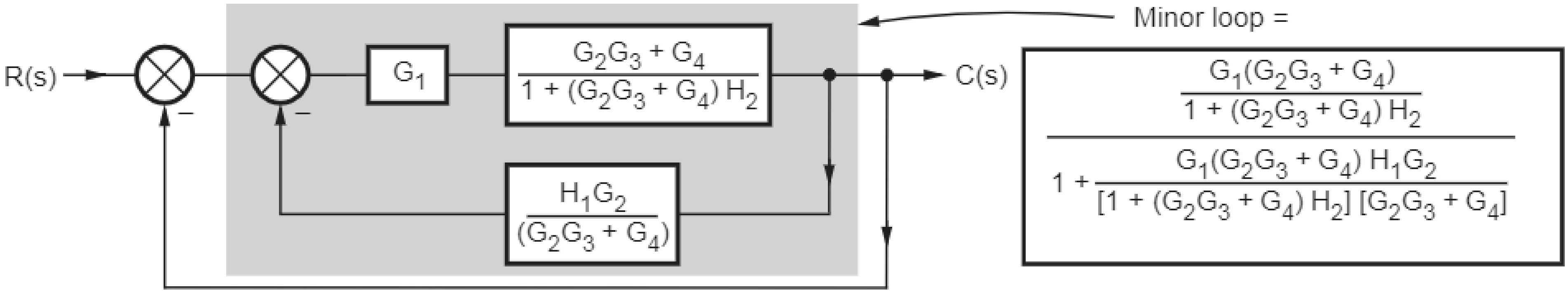
$$\frac{G_2G_3 + G_4}{1 + (G_2G_3 + G_4)\,H_2}$$

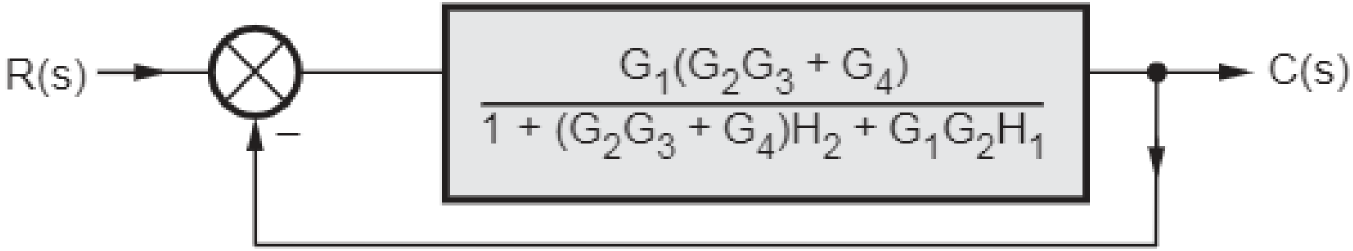


Minor loop =

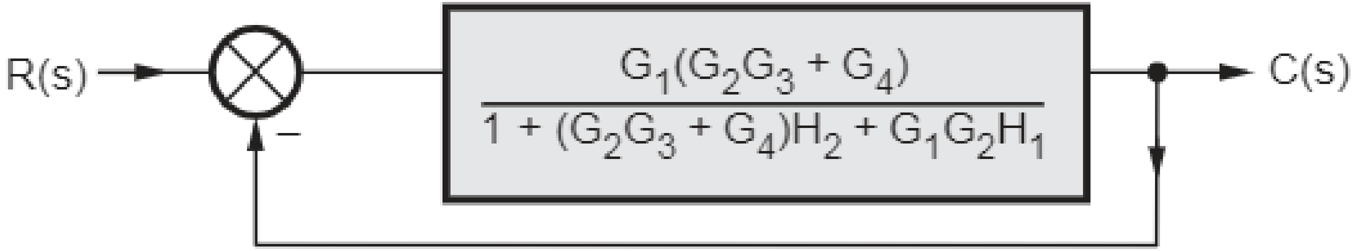
$$\frac{\dfrac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)\,H_2}}{1 + \dfrac{G_1(G_2G_3 + G_4)\,H_1G_2}{[1 + (G_2G_3 + G_4)\,H_2]\,[G_2G_3 + G_4]}}$$

Combine the blocks in cascade and eliminating feedback path





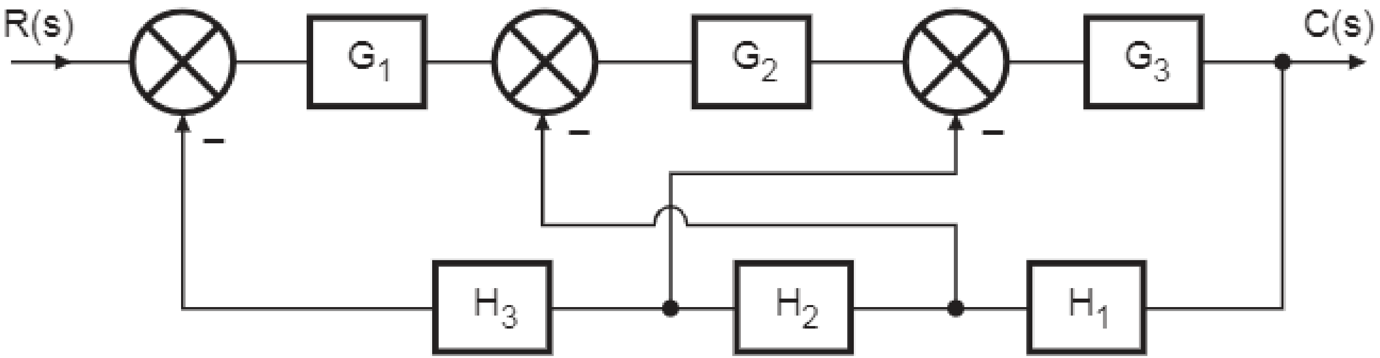
Eliminating feedback path



$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2+G_1G_2H_1}}$$

$$= \boxed{\frac{G_1G_2G_3+G_1G_4}{1+G_2G_3H_2+G_4H_2+G_1G_2H_1+G_1G_2G_3+G_1G_4}}$$

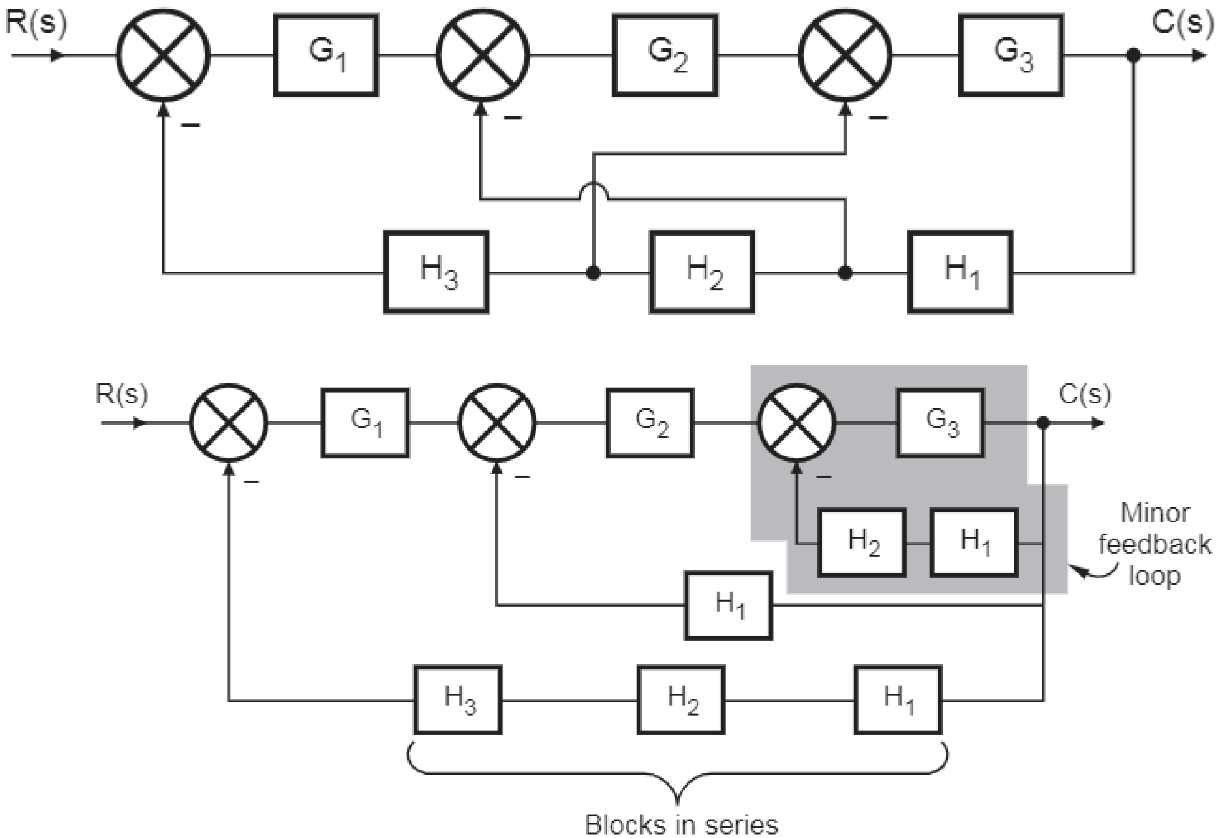
4.Reduce the block diagram shown in figure and find T.F C(S)/R(S).

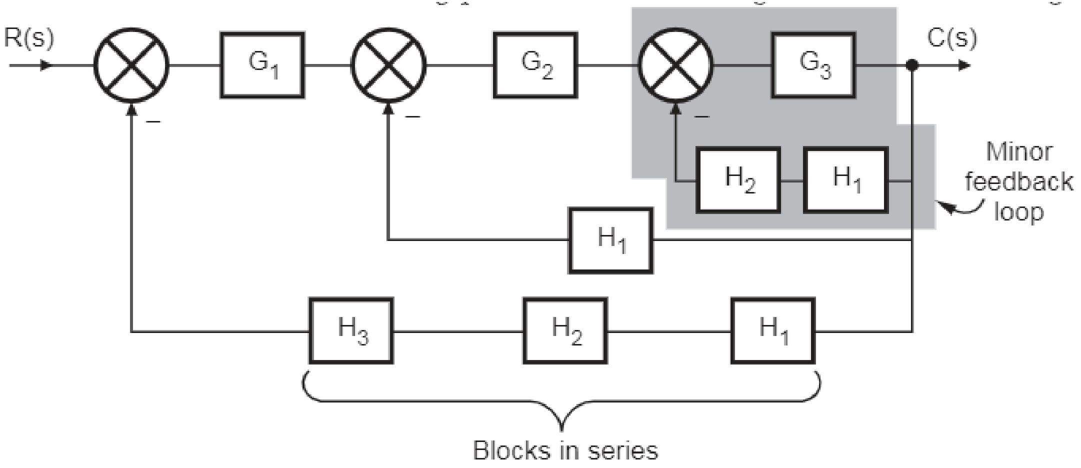


ATME College of Engineering

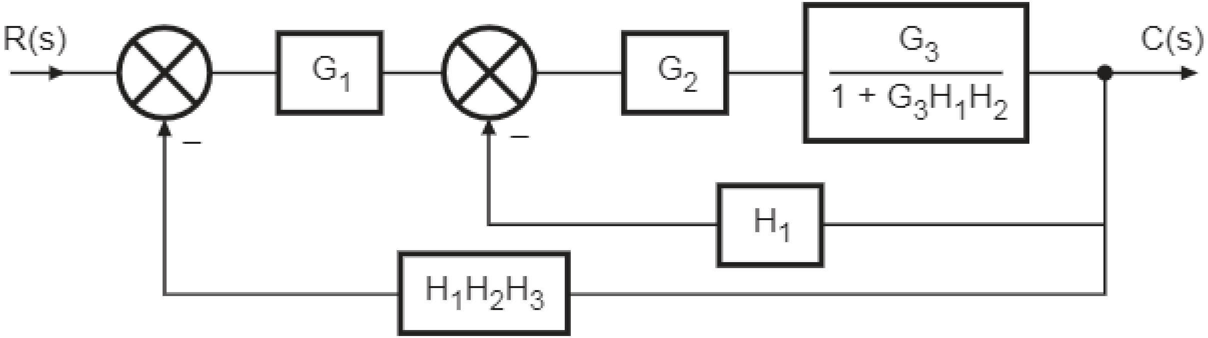
Department of EEE
Emitting Elite Energy

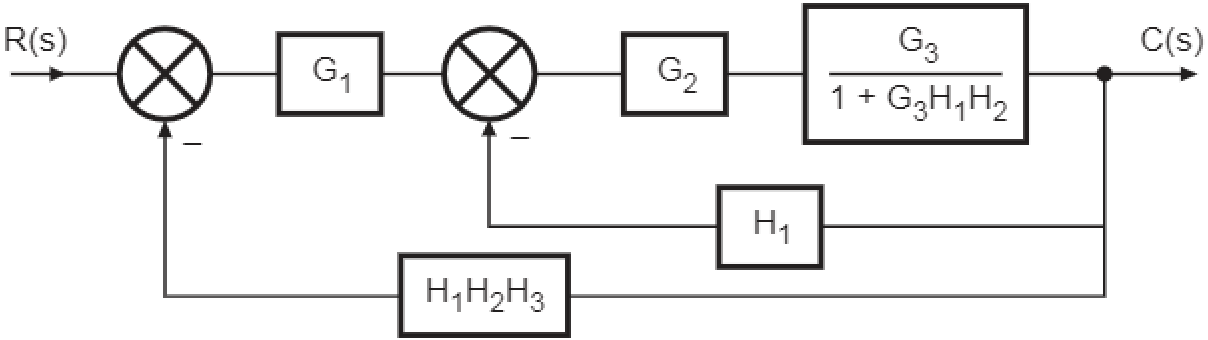
Moving the branch point/Take off point after the block



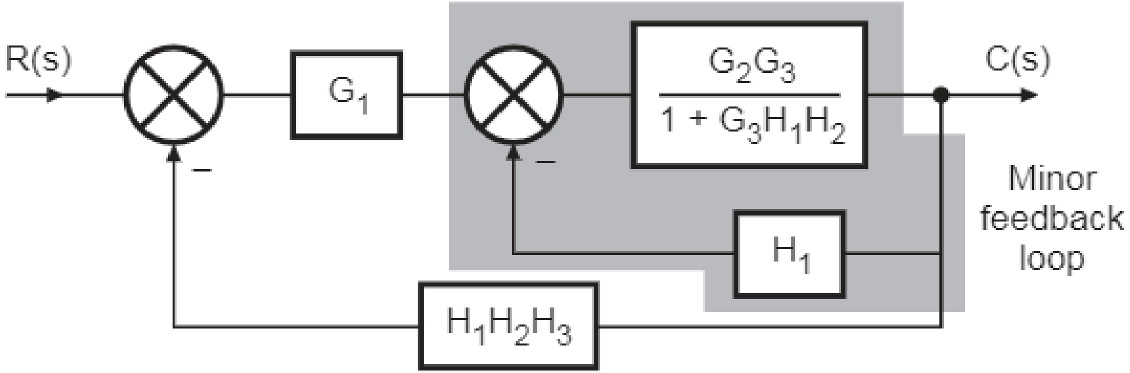


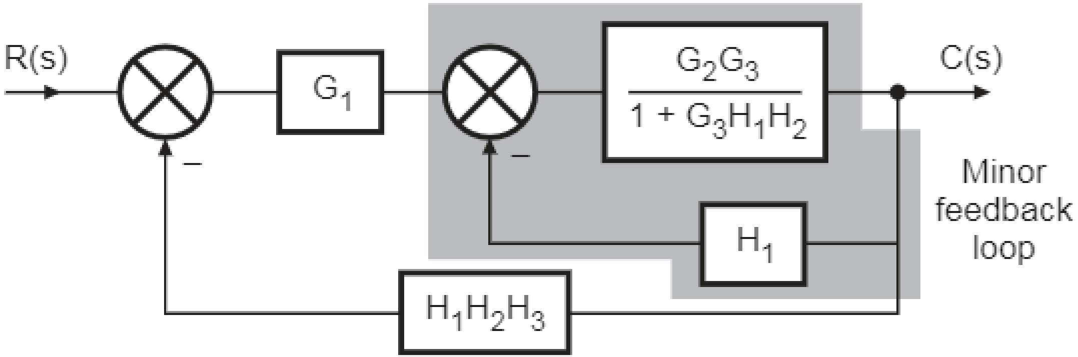
Eliminating feedback path & Combining blocks in cascade



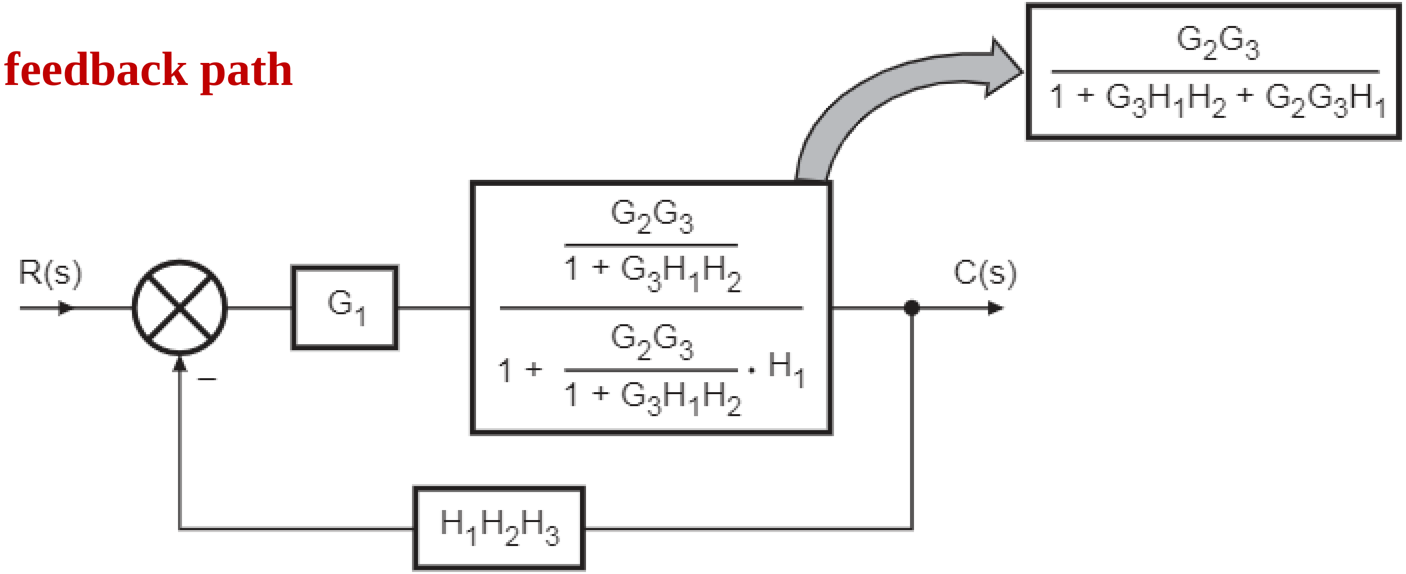


Combining blocks in cascade & Eliminating feedback path

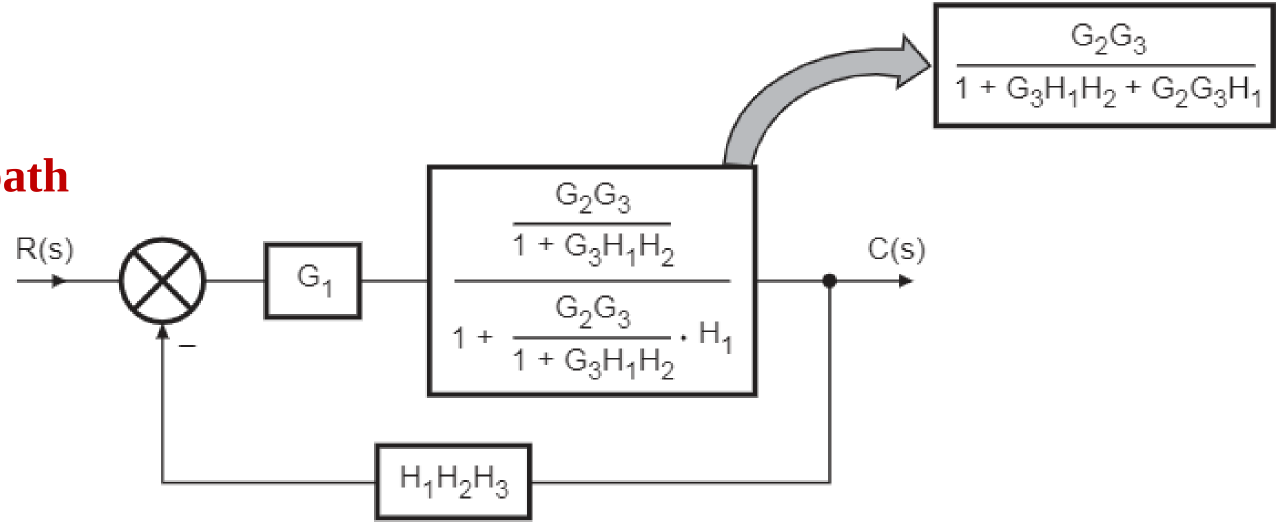


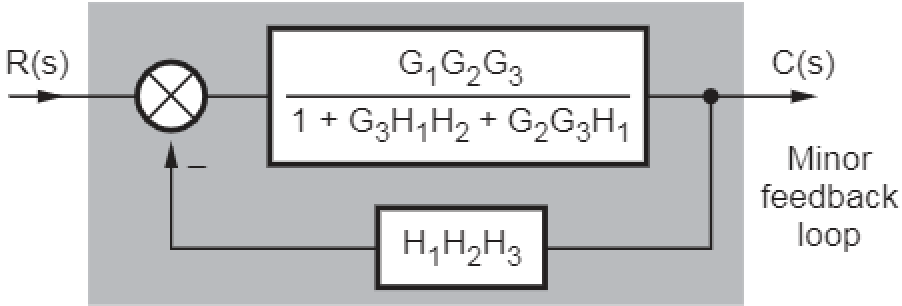


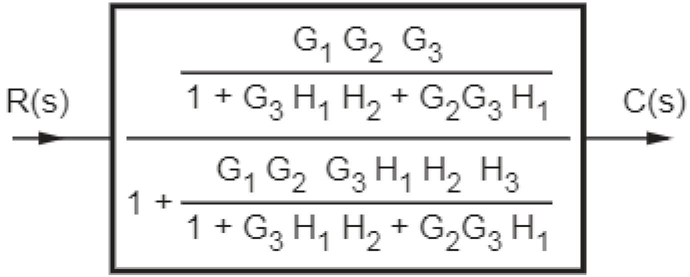
Eliminating feedback path



Eliminating feedback path

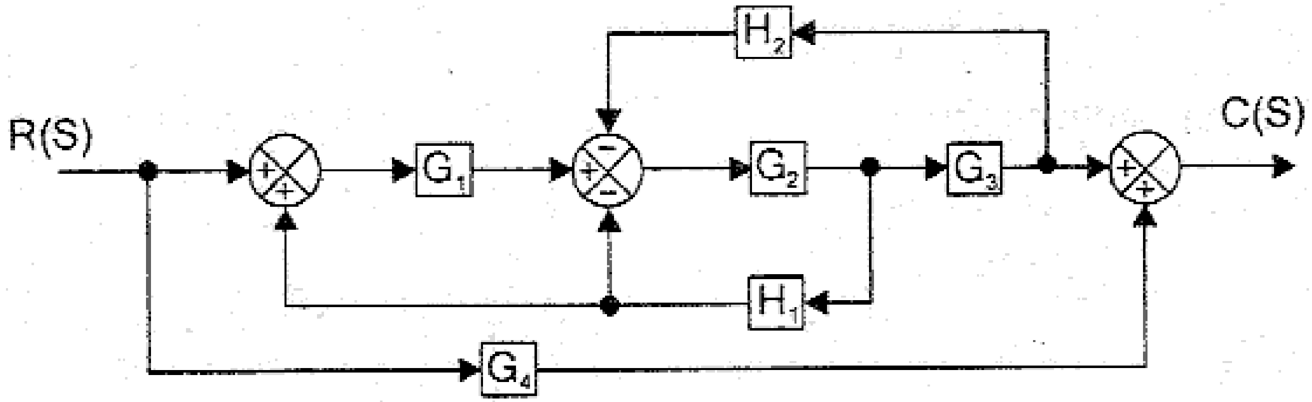




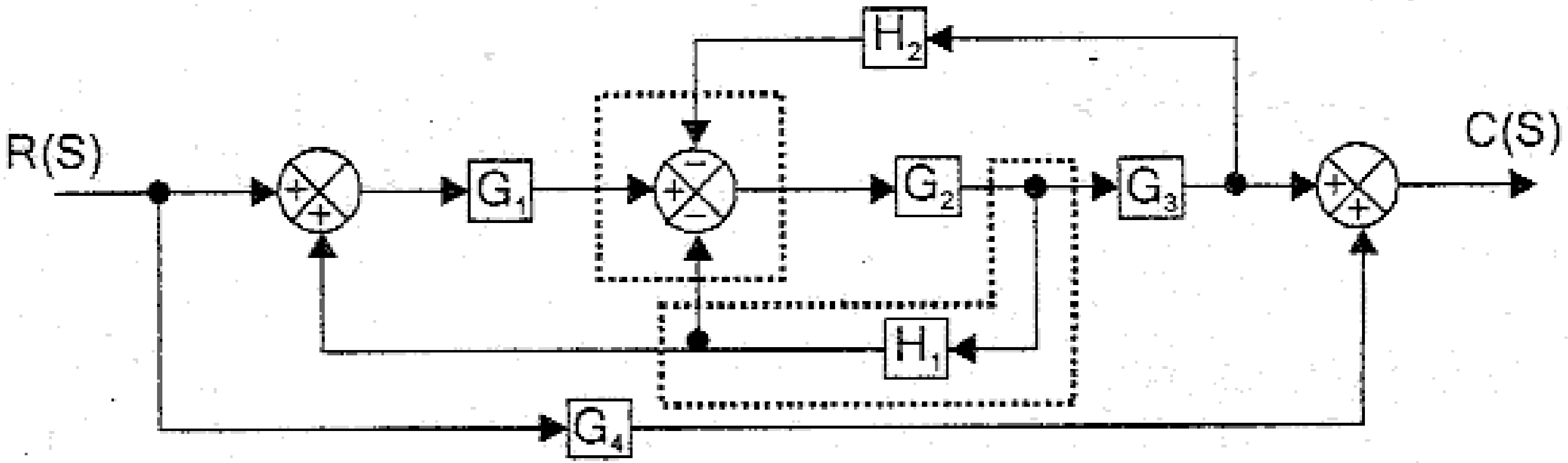


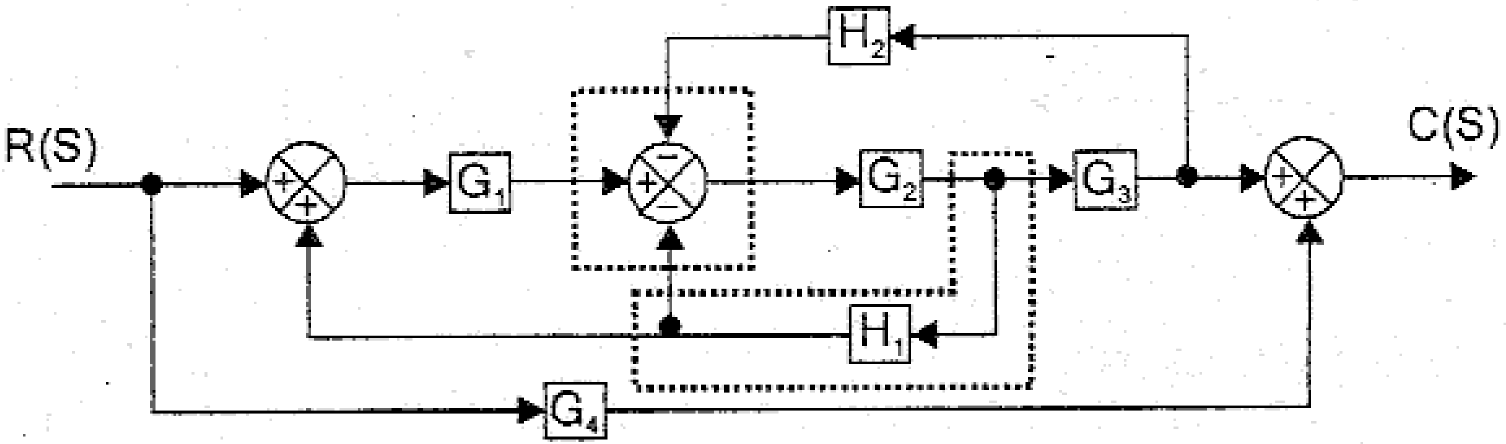
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1\ G_2\ G_3}{1 + G_3\ H_1\ H_2 + G_2\ G_3\ H_1 + G_1\ G_2\ G_3\ H_1\ H_2\ H_3}$$

5.Reduce the block diagram shown in figure and find T.F C(S)/R(S).

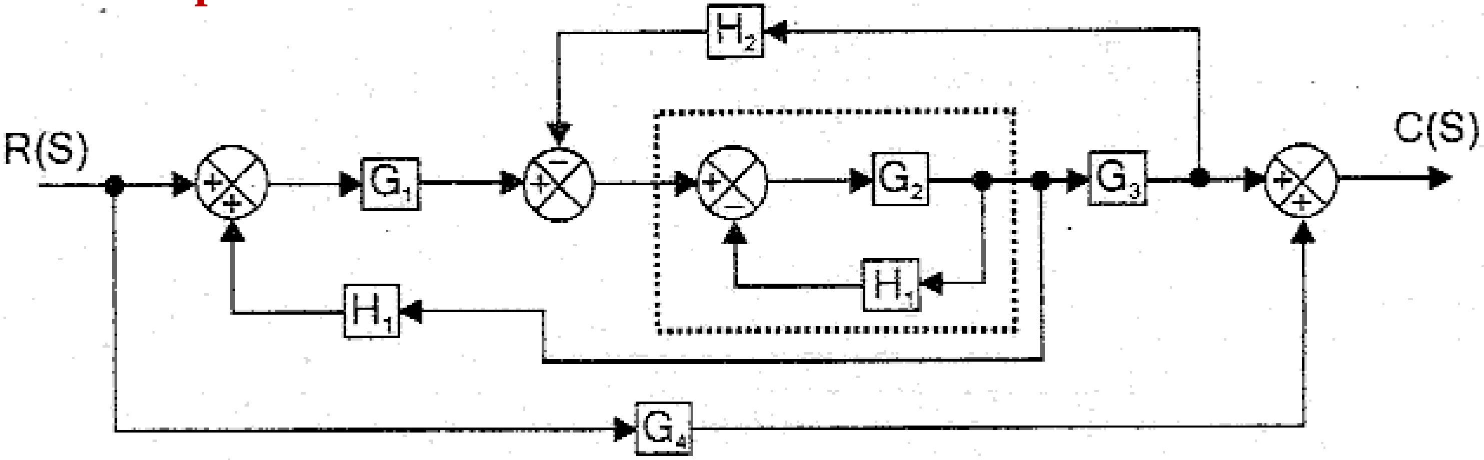


Splitting the summing point and rearranging the branch point

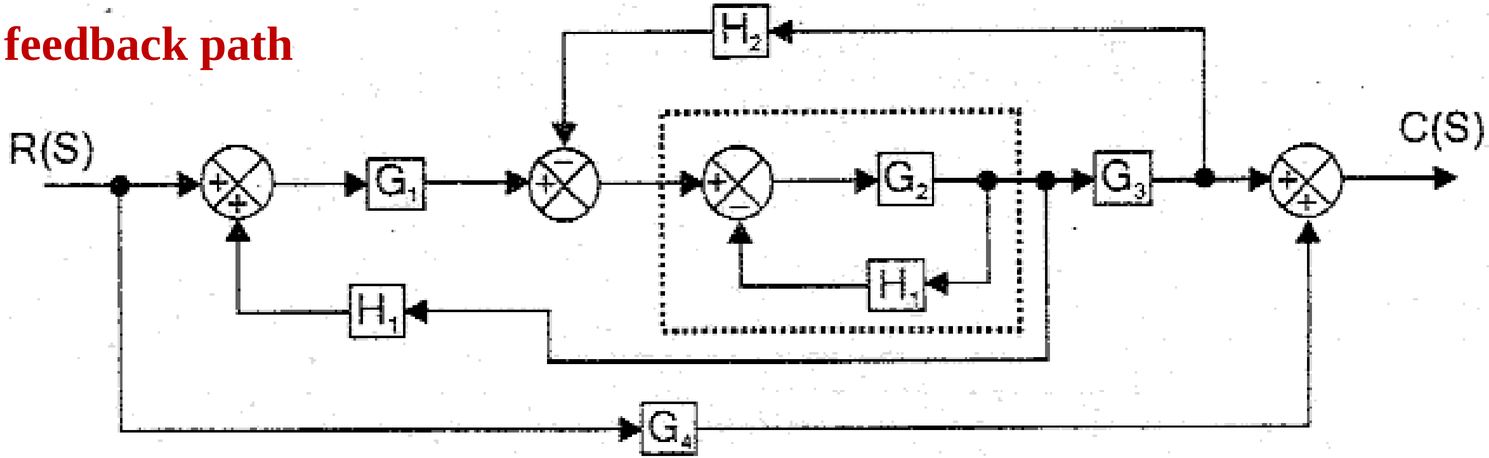




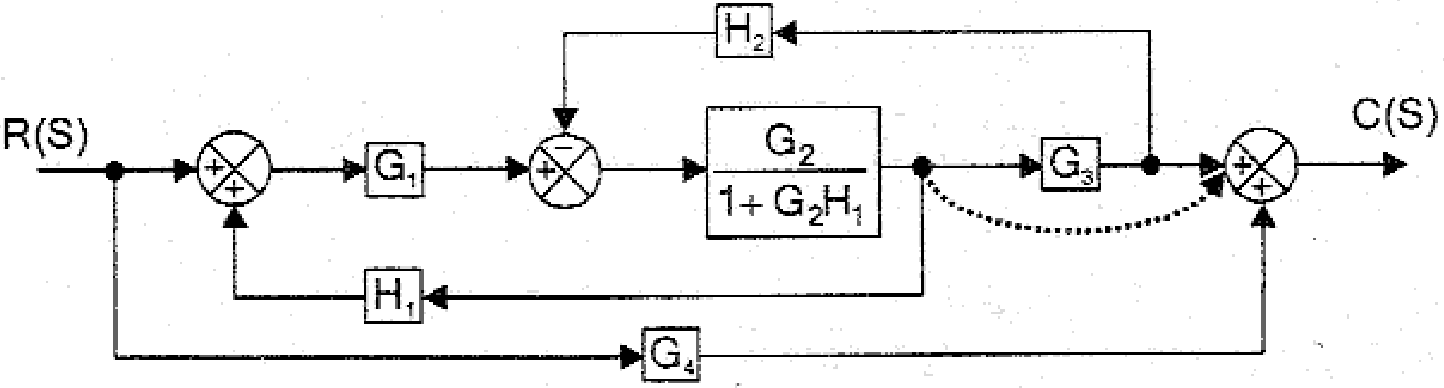
Eliminating feedback path



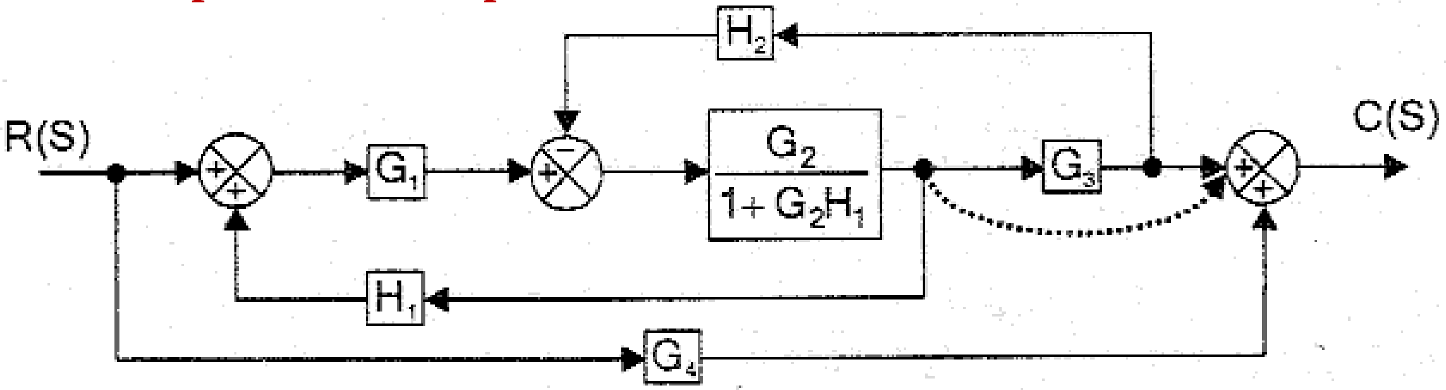
Eliminating feedback path



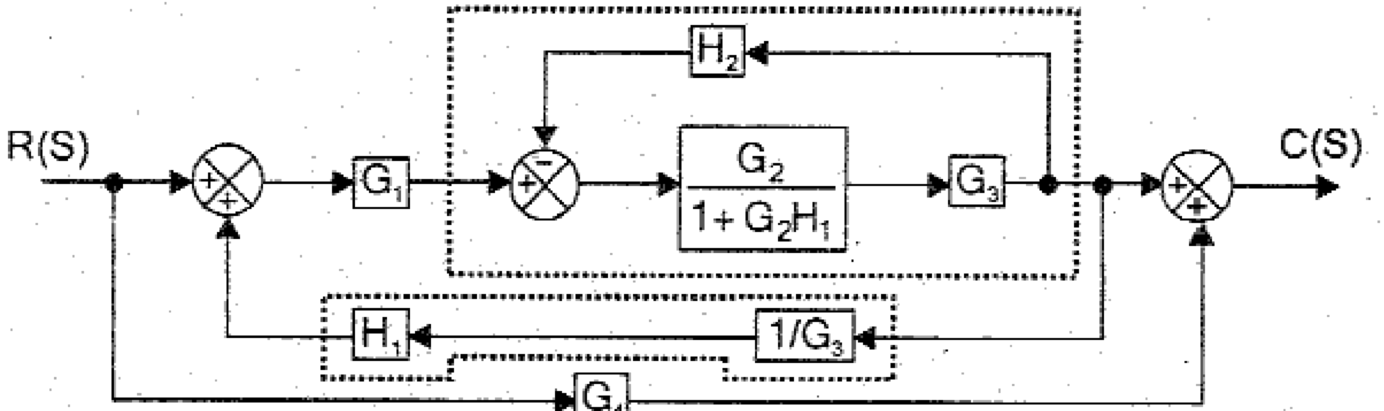
Shift the Branch point/take-off point after the block



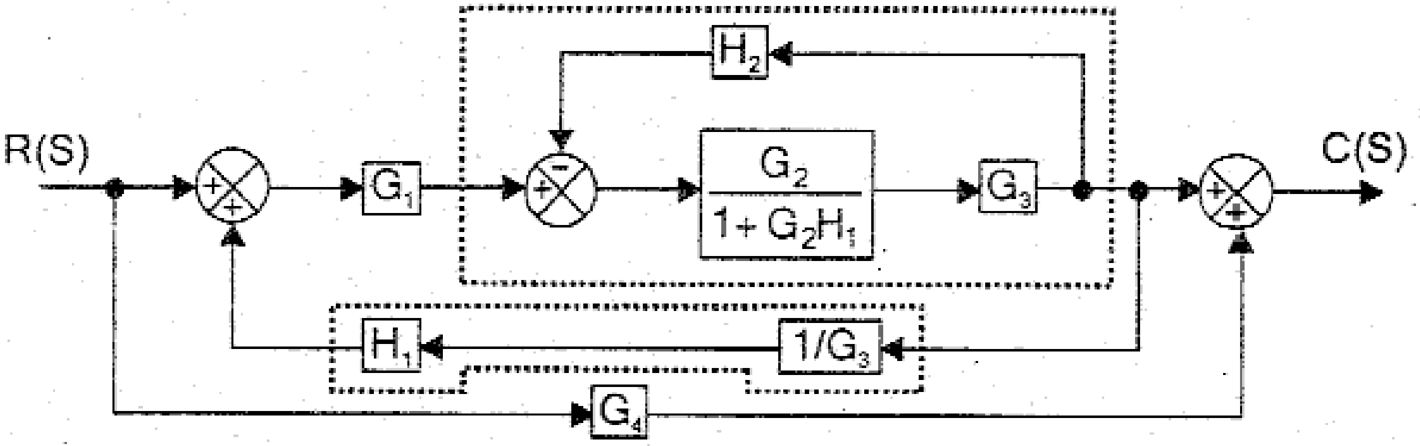
Shift the Branch point/take-off point after the block

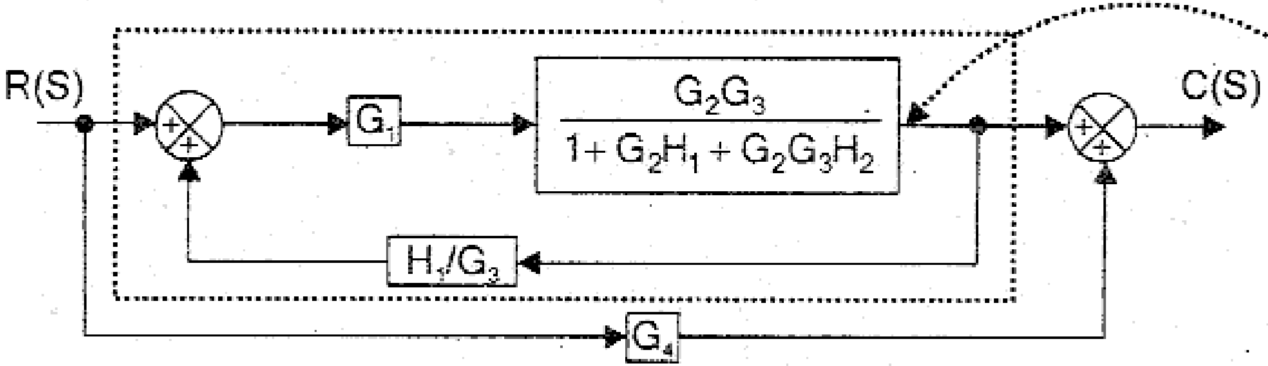


Combine the blocks in cascade and eliminating feedback path

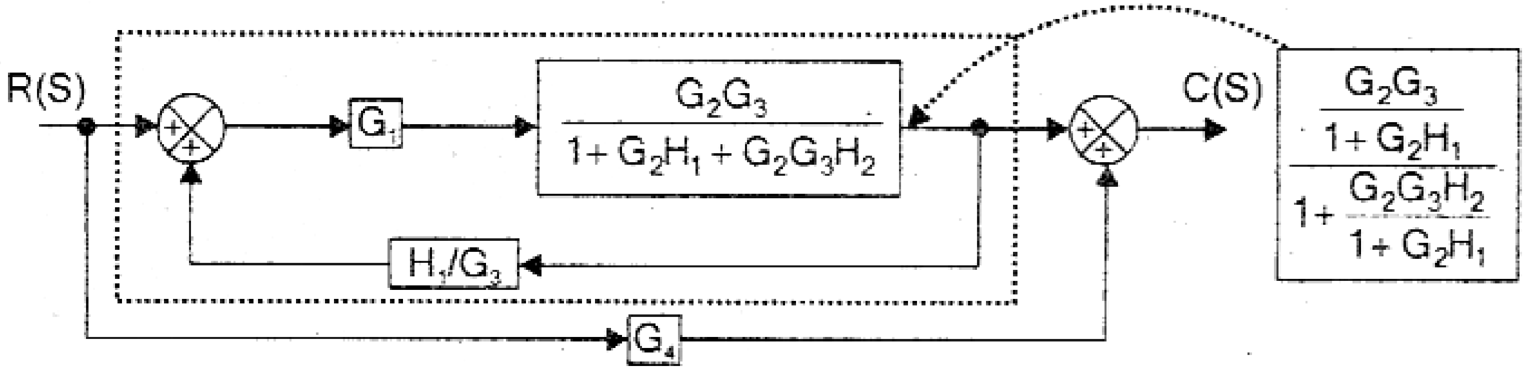


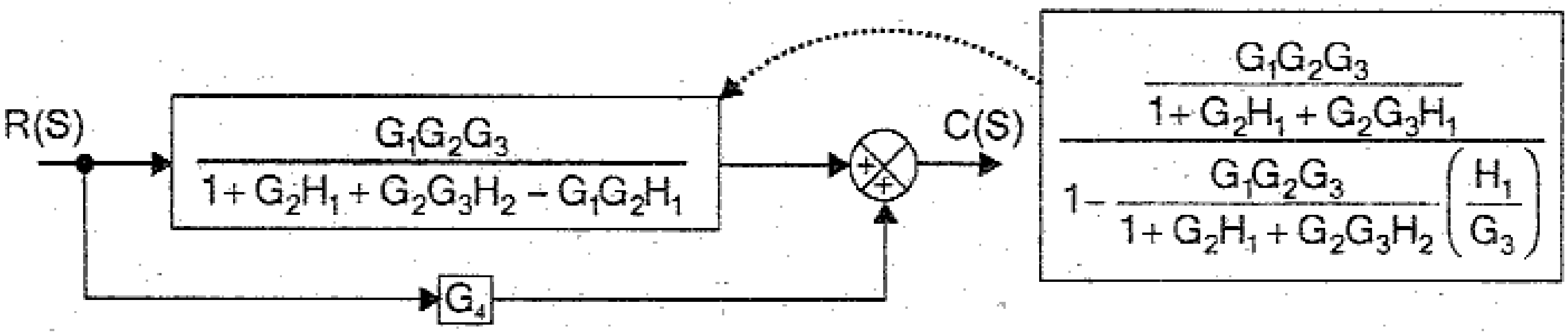
Combine the blocks in cascade and eliminating feedback path



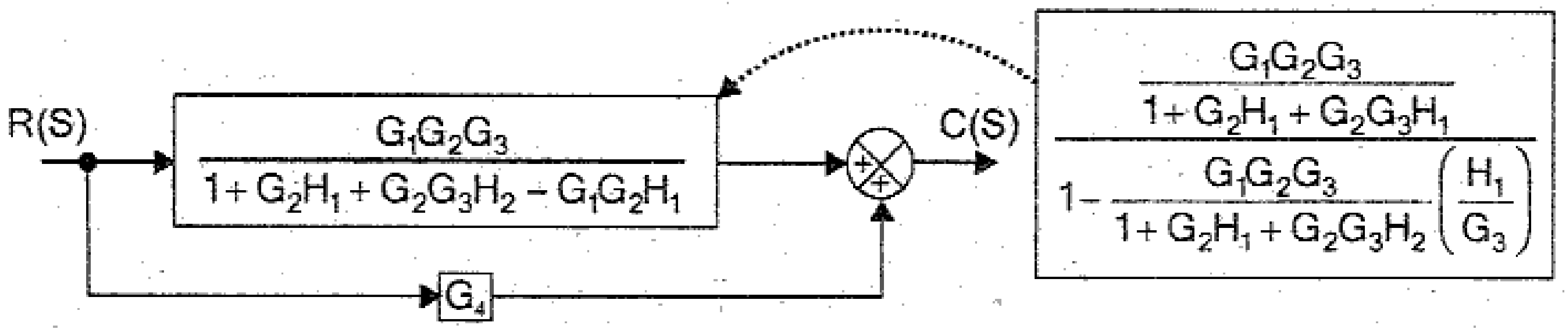


$$\frac{\dfrac{G_2G_3}{1+G_2H_1}}{1+\dfrac{G_2G_3H_2}{1+G_2H_1}}$$


R(S)
G_1
$\frac{G_2G_3}{1+G_2H_1+G_2G_3H_2}$
H_1/G_3
G_4
C(S)
$\frac{\frac{G_2G_3}{1+G_2H_1}}{1+\frac{G_2G_3H_2}{1+G_2H_1}}$


R(S)
$\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1}$
G_4
C(S)
$\frac{\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_1}}{1-\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2}\left(\frac{H_1}{G_3}\right)}$

Combining parallel blocks / Eliminating forward path

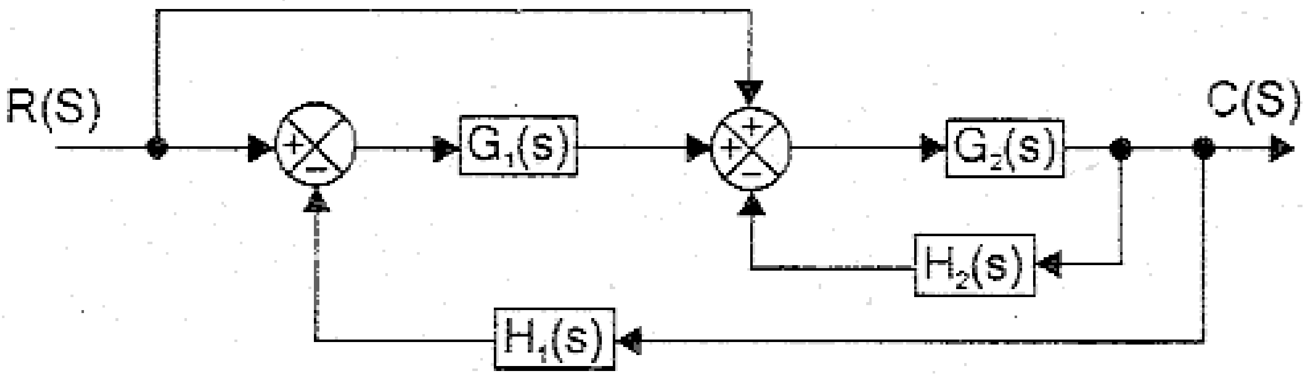


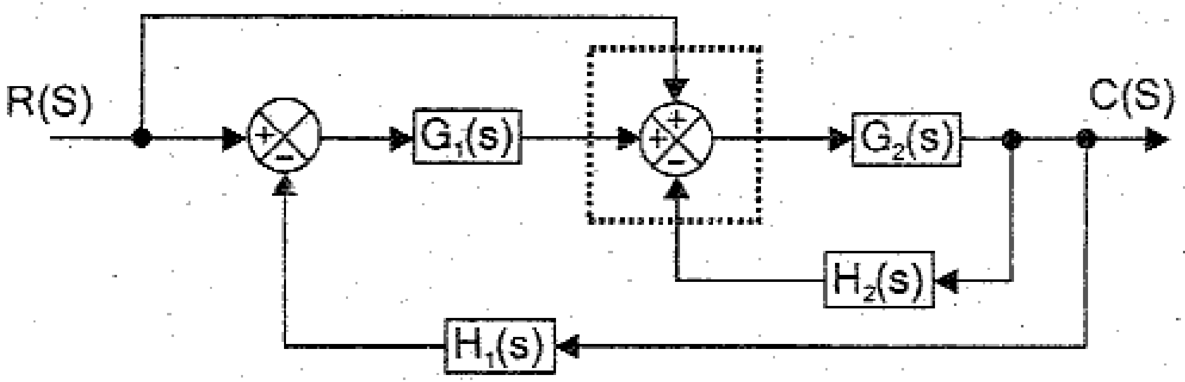
$$\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1}$$

$$\frac{\dfrac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_1}}{1-\dfrac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2}\left(\dfrac{H_1}{G_3}\right)}$$

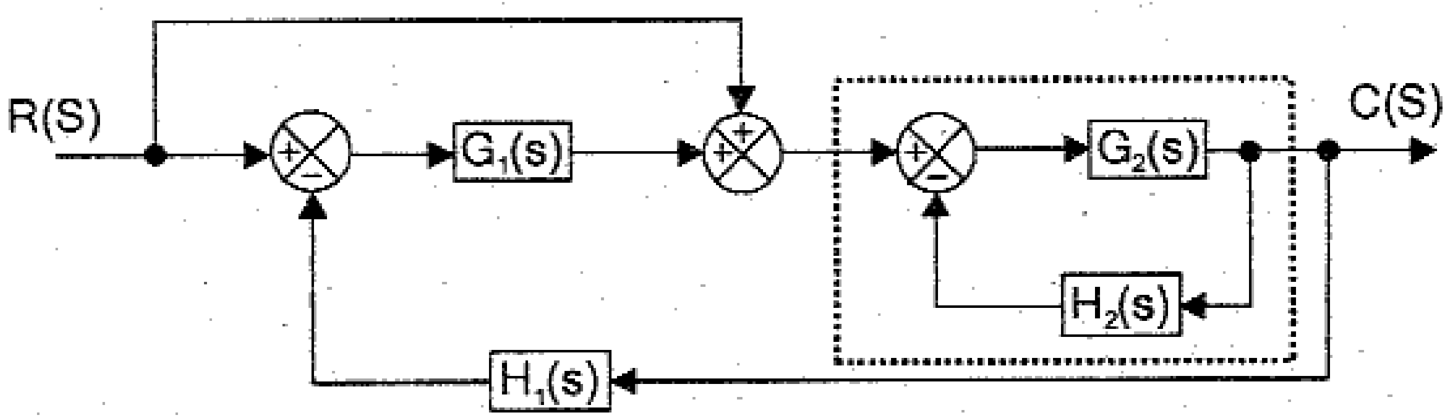
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1} + G_4$$

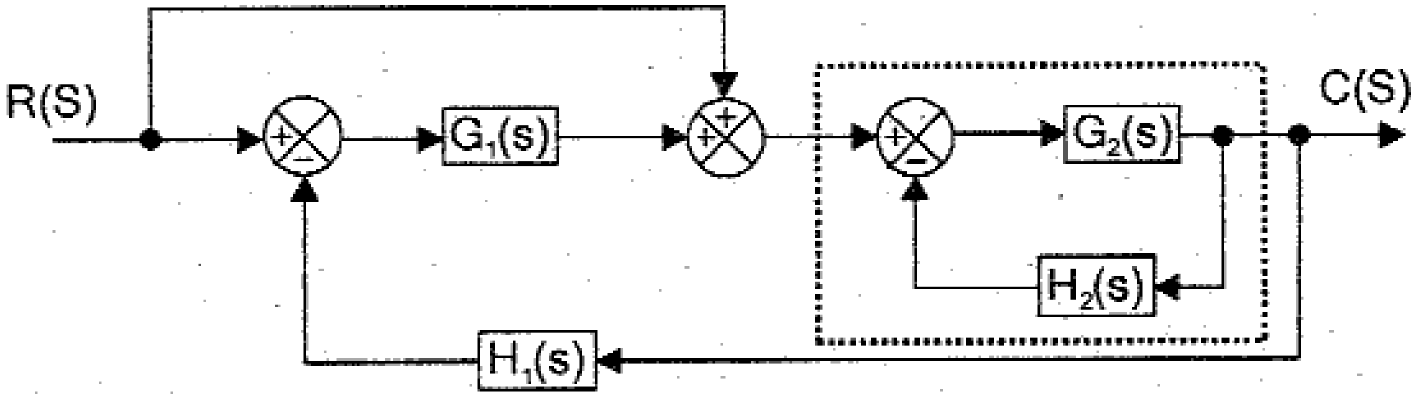
6.Reduce the block diagram shown in figure and find T.F C(S)/R(S).



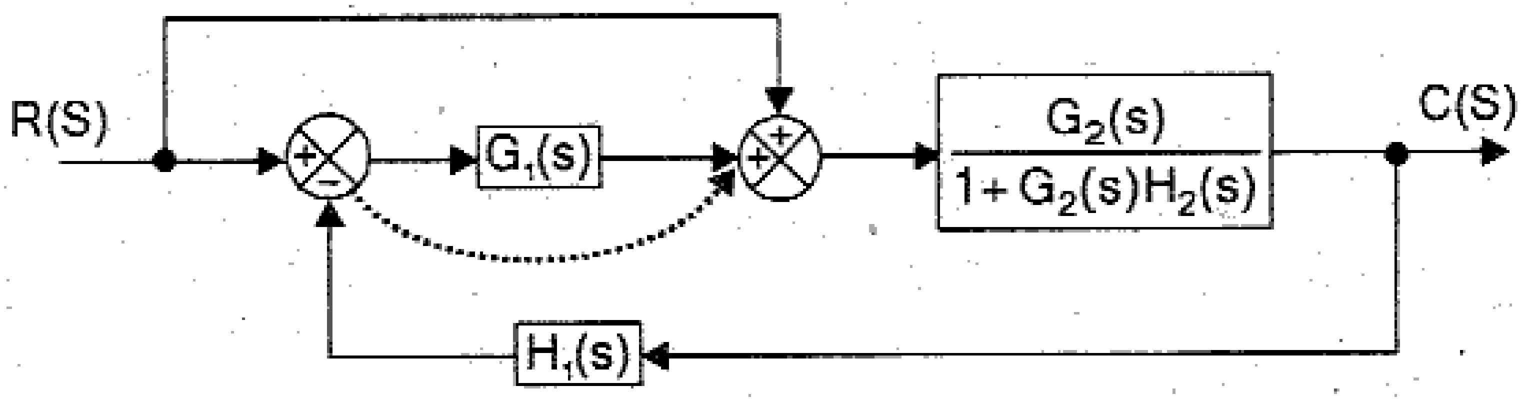


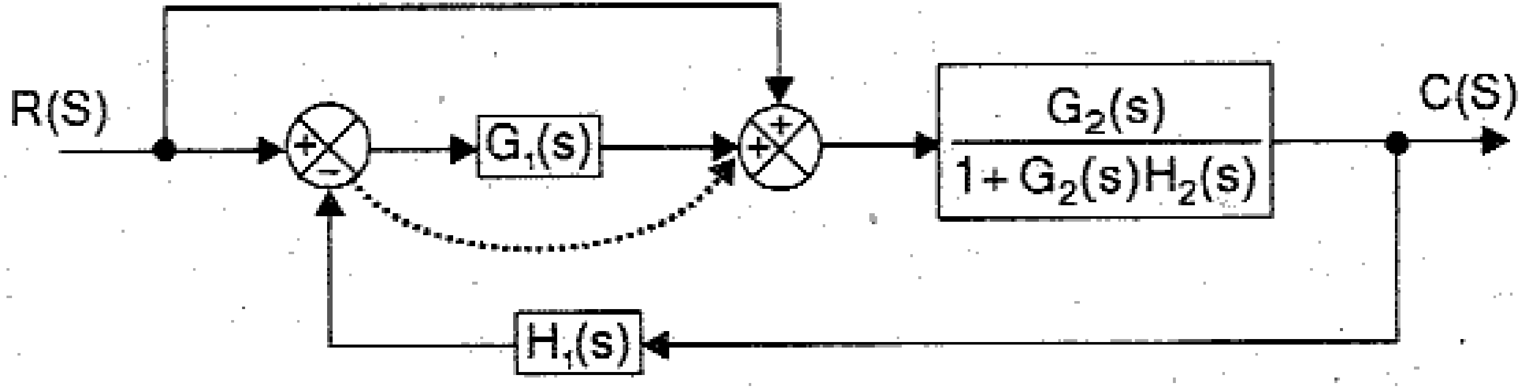
Splitting the summing point and Eliminating feedback path



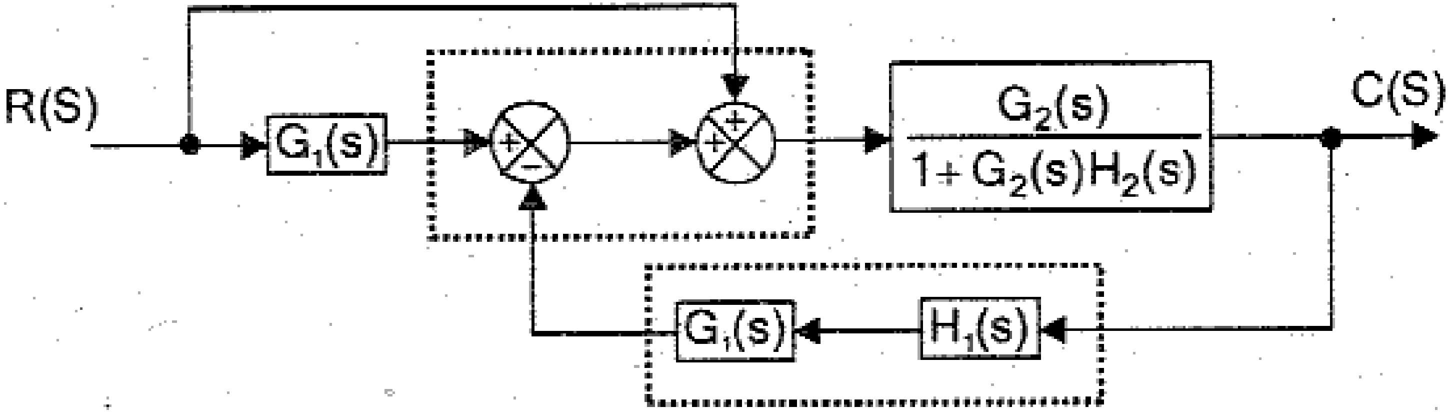


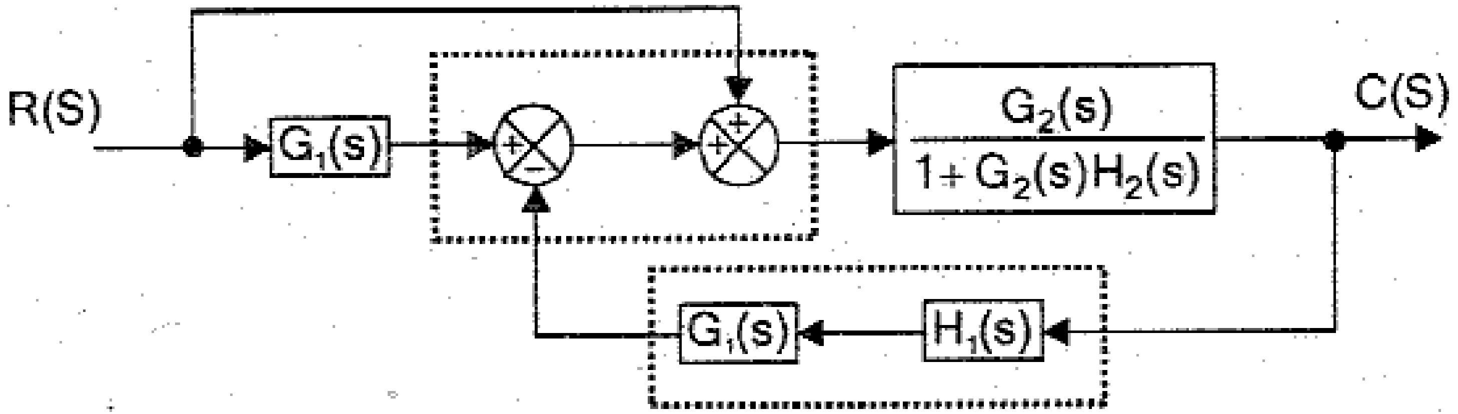
Moving the summing point after the block



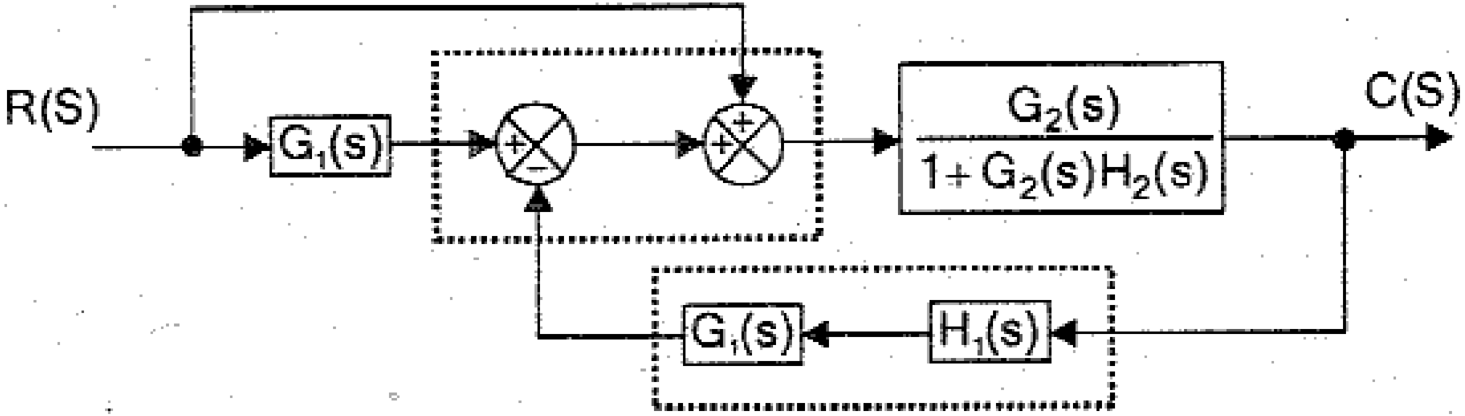


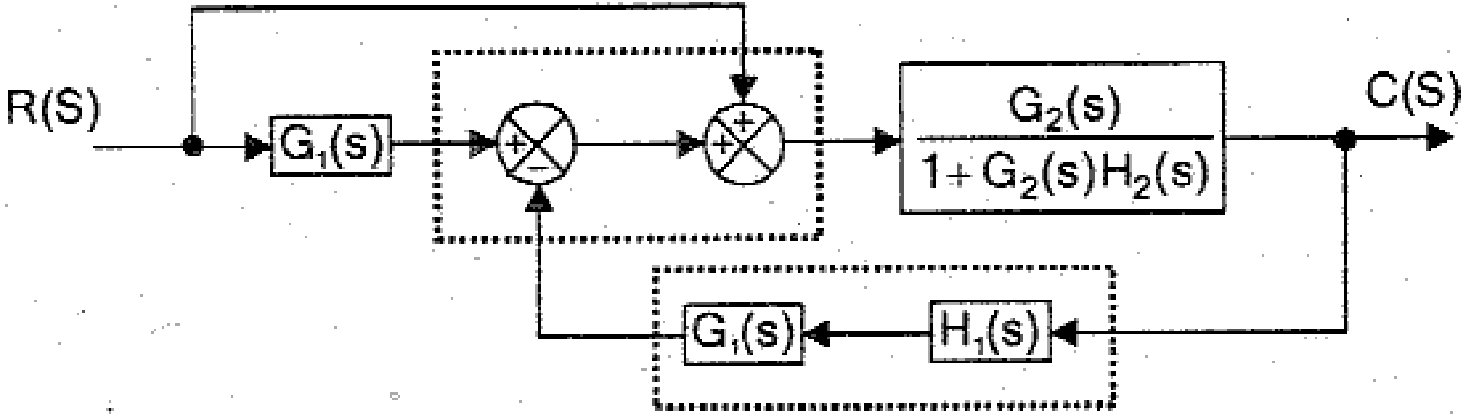
Moving the summing point after the block



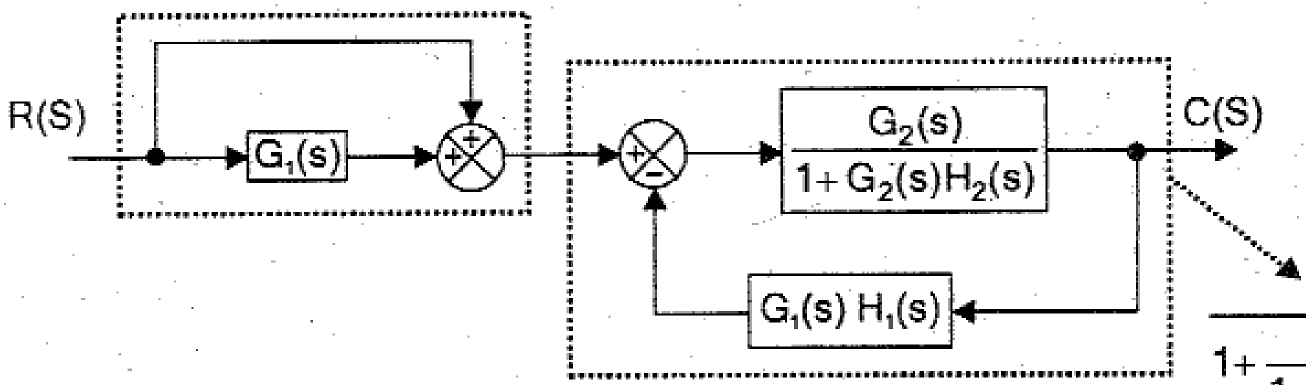


Interchanging the summing point and Combine the blocks in cascade

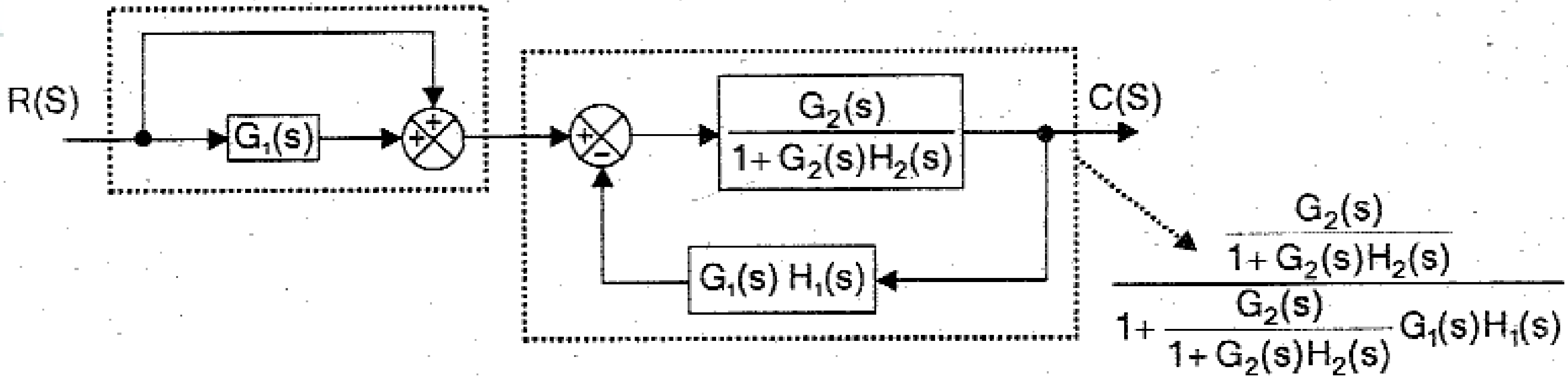




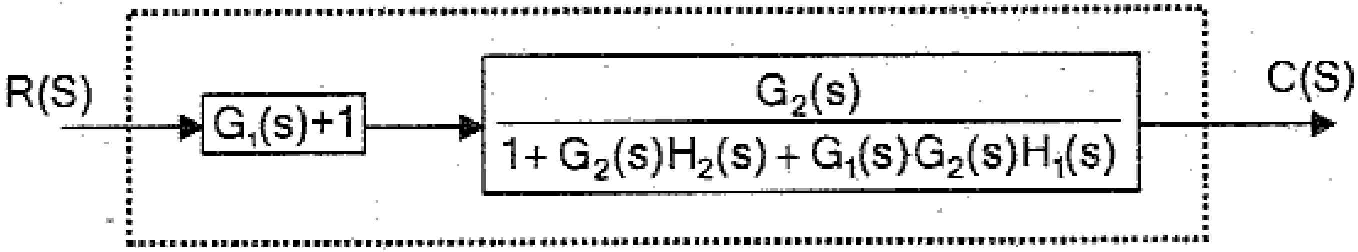
Eliminating the Feedback path and feed forward path/Parallel block

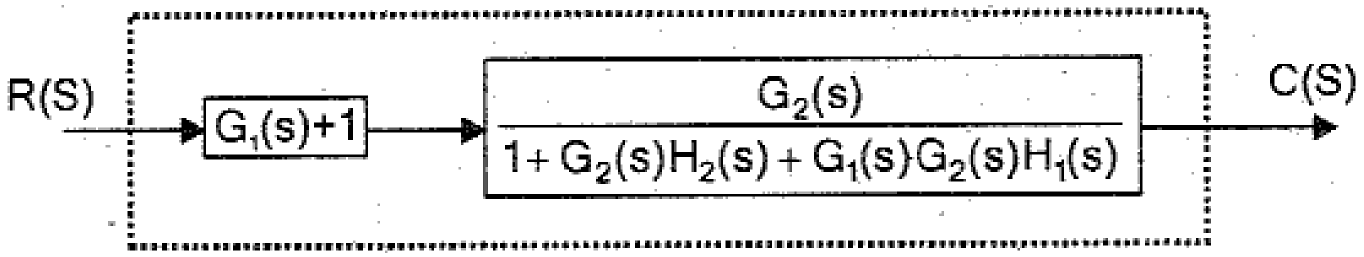


$$\frac{\dfrac{G_2(s)}{1+G_2(s)H_2(s)}}{1+\dfrac{G_2(s)}{1+G_2(s)H_2(s)}G_1(s)H_1(s)}$$



Eliminating the Feedback path and feed forward path/Parallel block

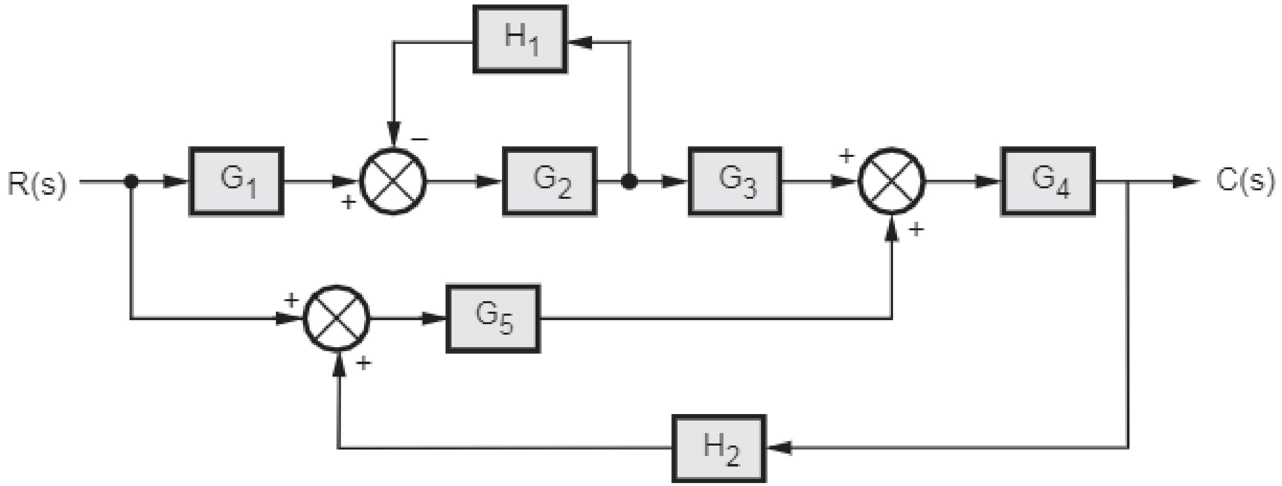


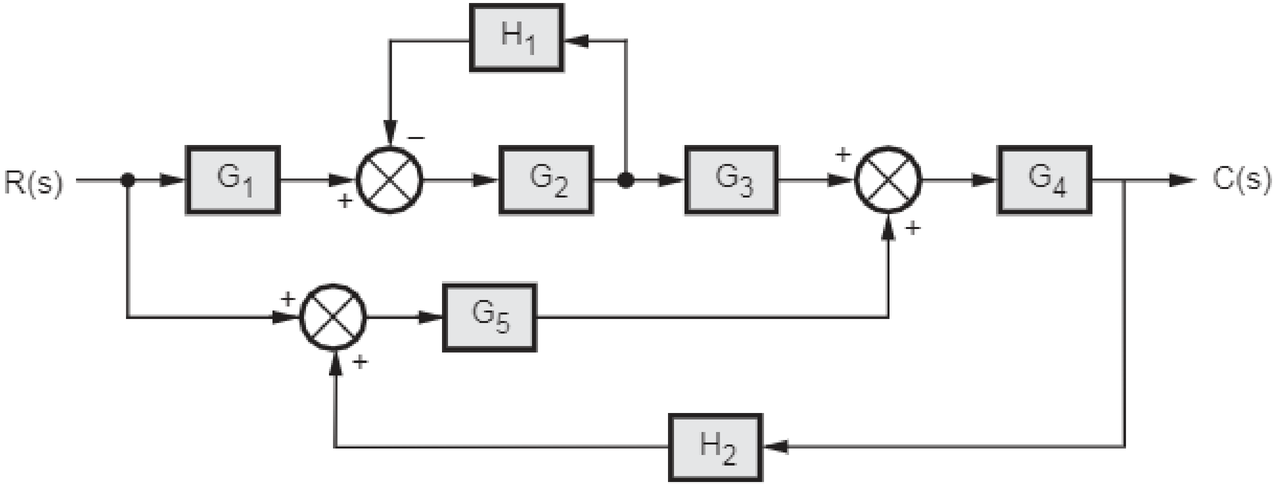


Combine the blocks in cascade

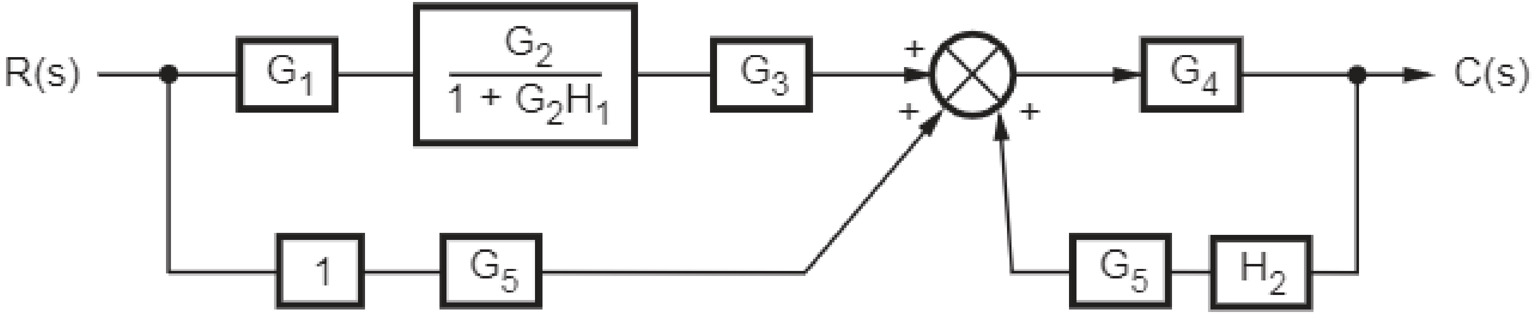
$$\therefore \frac{C(s)}{R(s)} = \frac{G_2(s)\,[G_1(s)+1]}{1+G_2(s)\,H_2(s)+G_1(s)\,G_2(s)\,H_1(s)}$$

7.Reduce the block diagram shown in figure and find T.F C(S)/R(S).

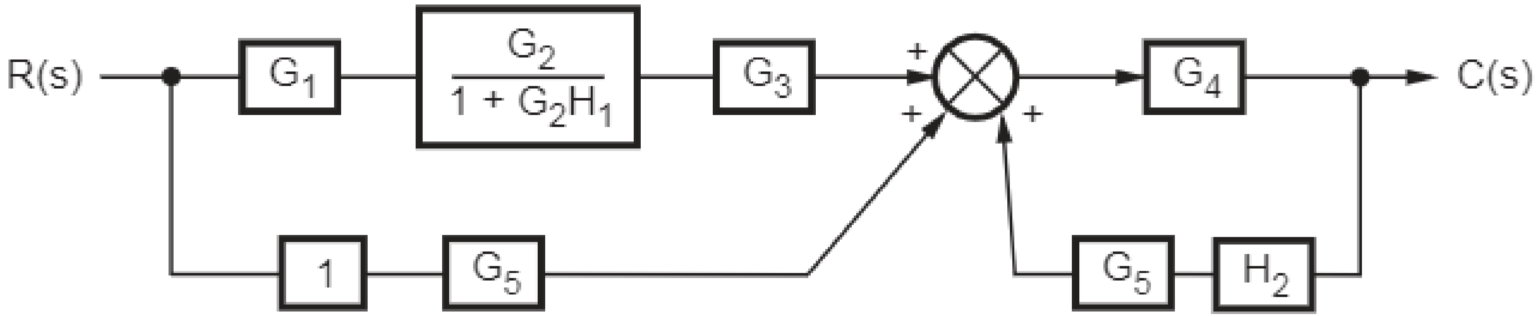


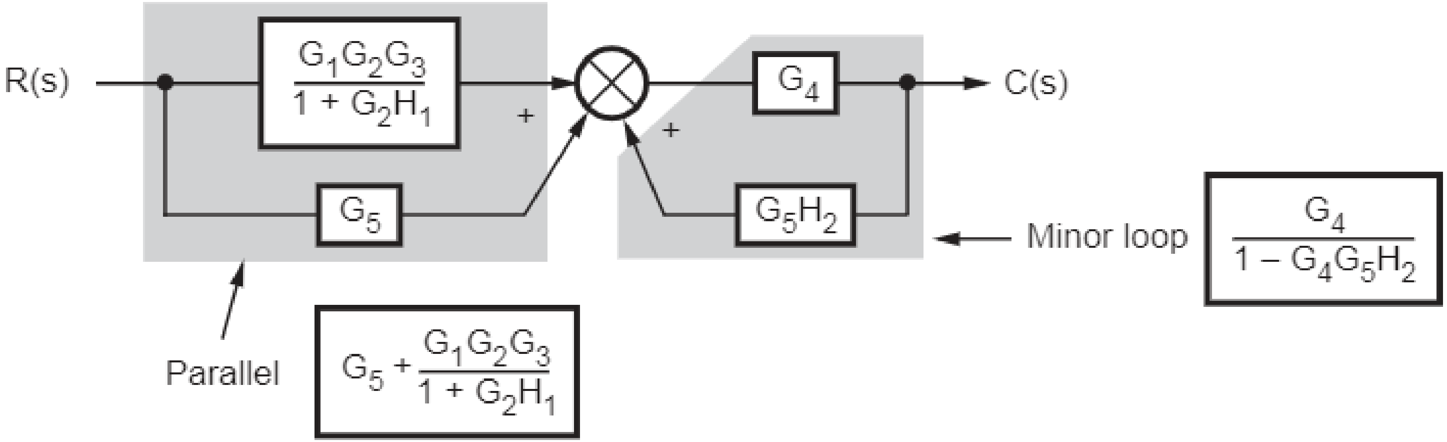


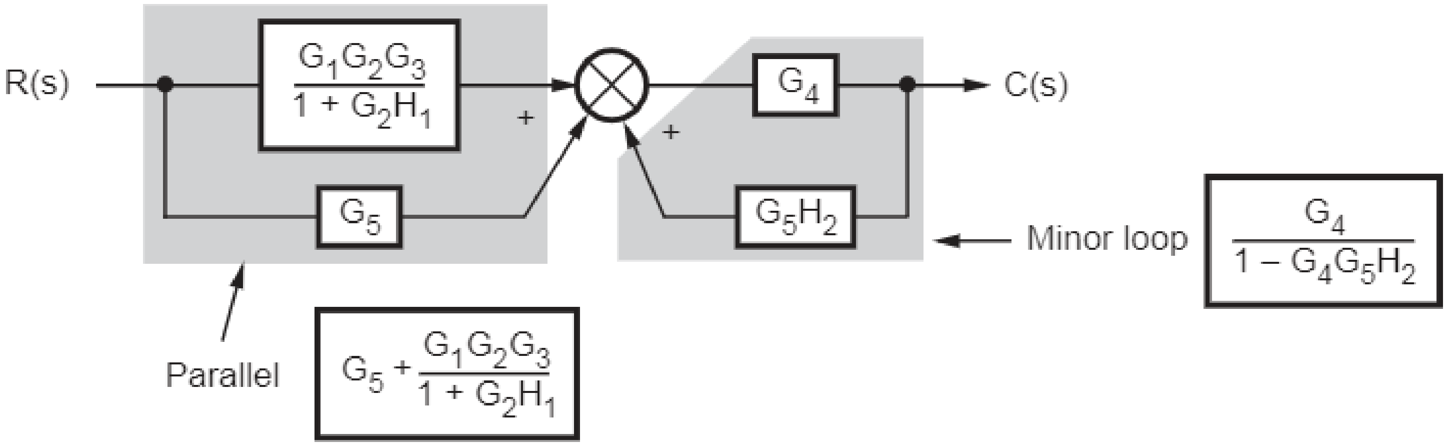
Separate the path through summing points in feedback path and reduce minor loop of G2 and H1



Separate the path through summing points in feedback path and reduce minor loop of G2 and H1





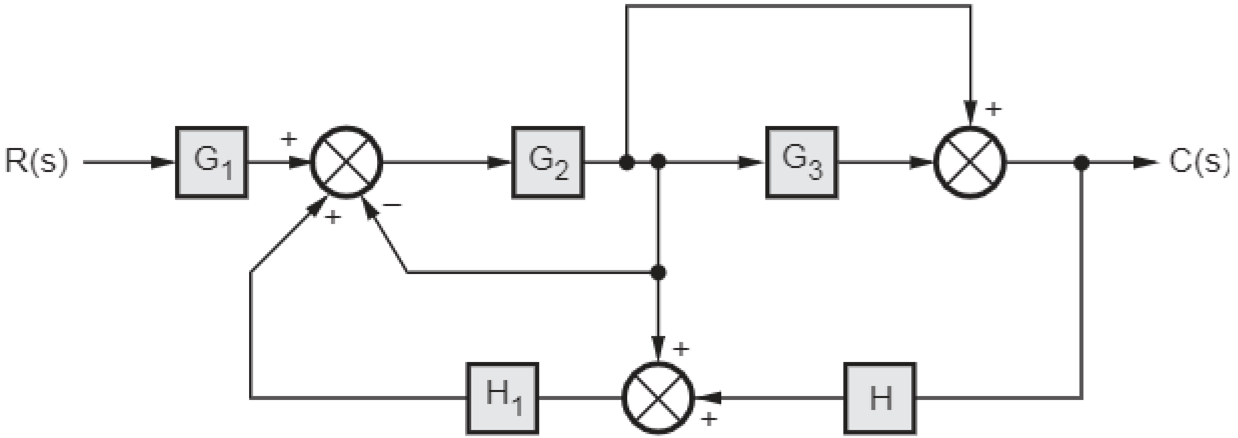


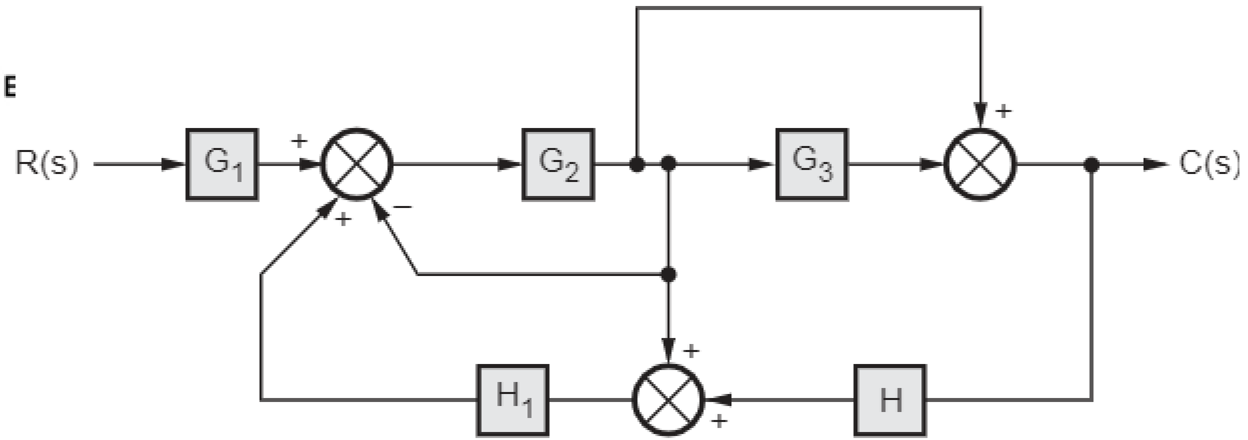
The shaded blocks are in series hence,

$$\frac{C(s)}{R(s)} = \left(G_5 + \frac{G_1G_2G_3}{1+G_2H_1}\right)\left(\frac{G_4}{1-G_4G_5H_2}\right)$$

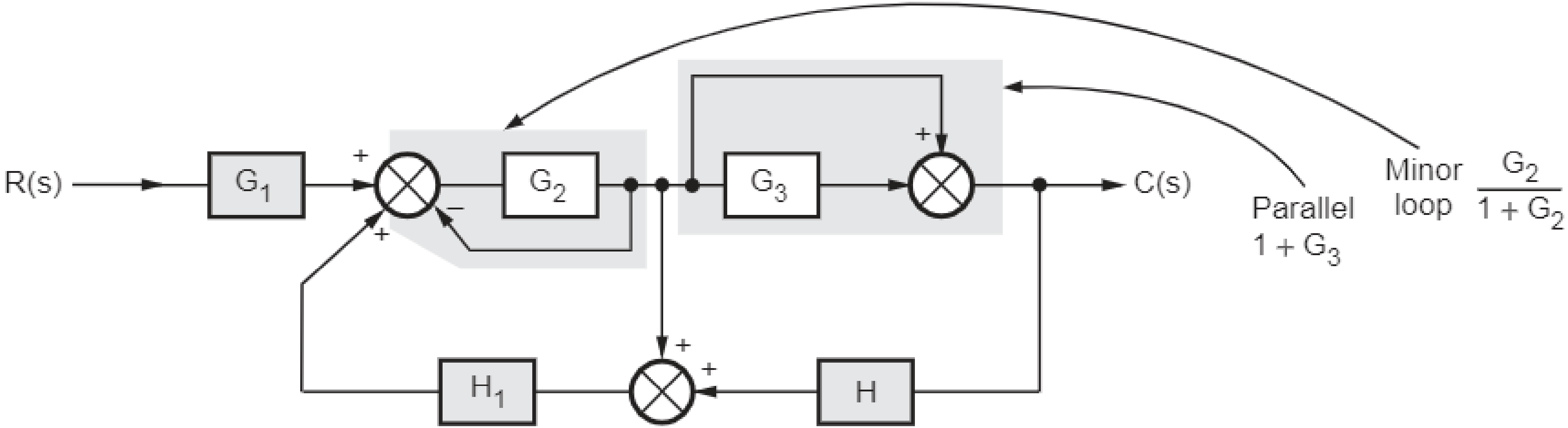
$$\frac{C(s)}{R(s)} = \boxed{\frac{G_4G_5 + G_2G_4G_5H_1 + G_1G_2G_3G_4}{1+G_2H_1 - G_4G_5H_2 - G_2G_4G_5H_1H_2}}$$

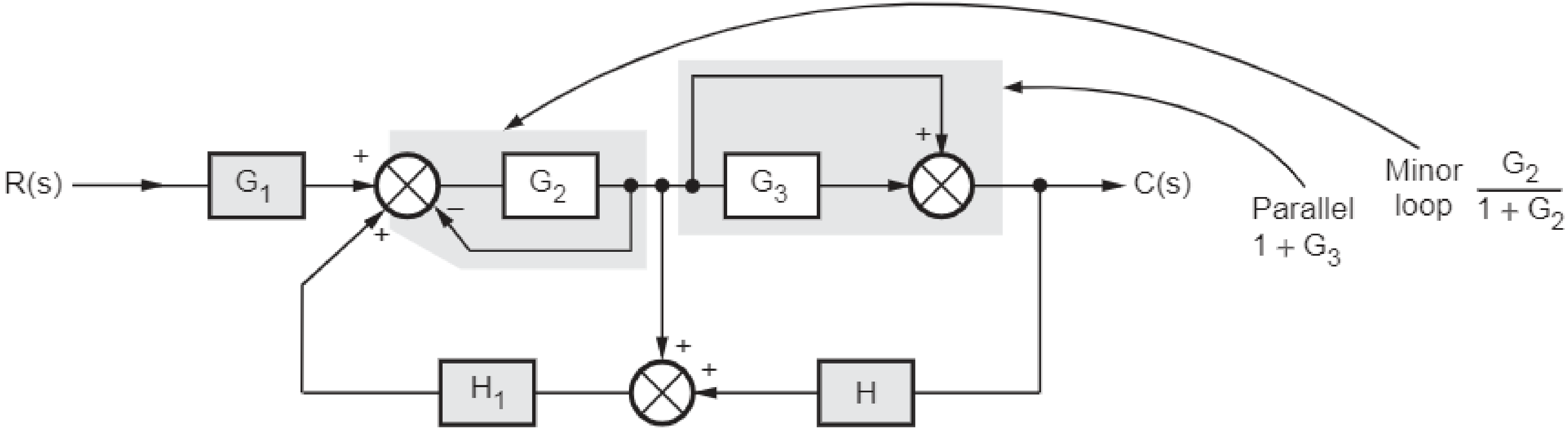
8.Reduce the block diagram shown in figure and find T.F C(S)/R(S).



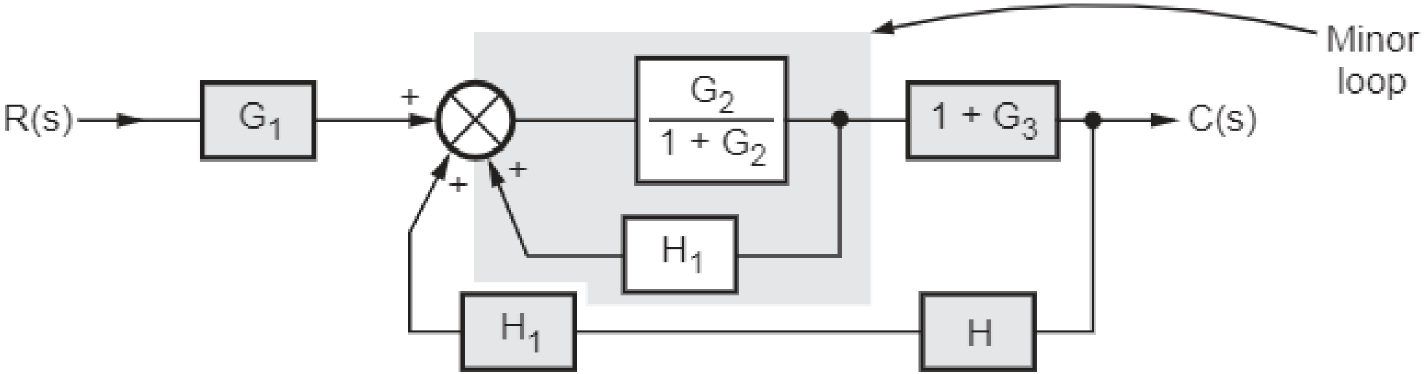


Separate the path linked through summing points of feedback path

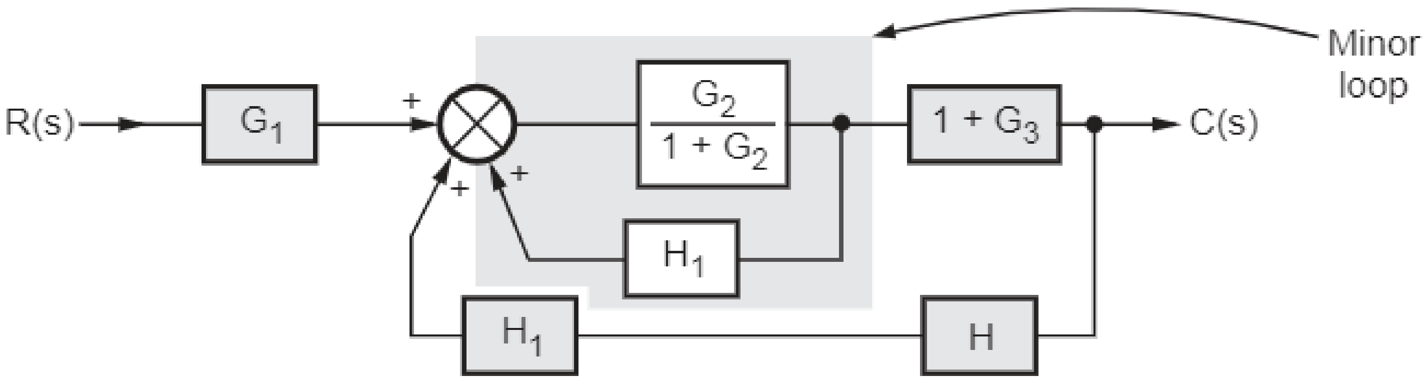




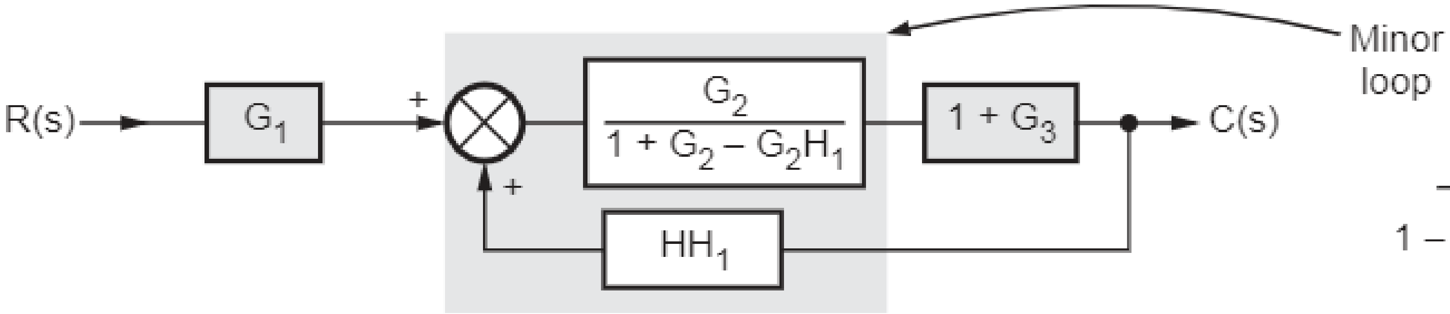
Separate the path linked through summing points of feedback path



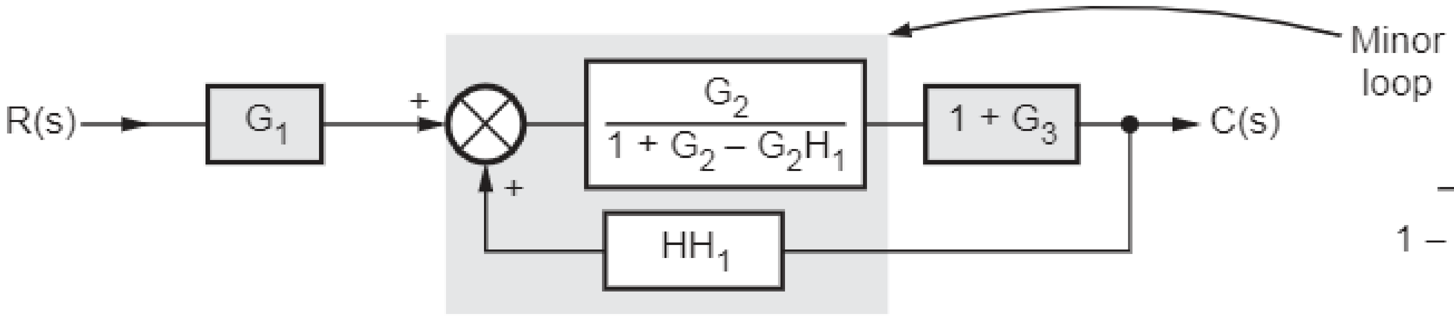
$$\frac{\dfrac{G_2}{1+G_2}}{1-\dfrac{G_2}{1+G_2}\times H_1} = \frac{G_2}{1+G_2-G_2H_1}$$



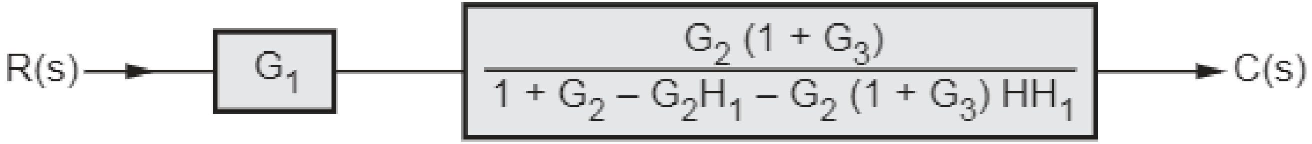
$$\frac{\dfrac{G_2}{1+G_2}}{1-\dfrac{G_2}{1+G_2}\times H_1} = \frac{G_2}{1+G_2-G_2H_1}$$



$$\frac{\dfrac{G_2(1+G_3)}{1+G_2-G_2H_1}}{1-\dfrac{G_2\,(1+G_3)\,HH_1}{1+G_2-G_2H_1}}$$

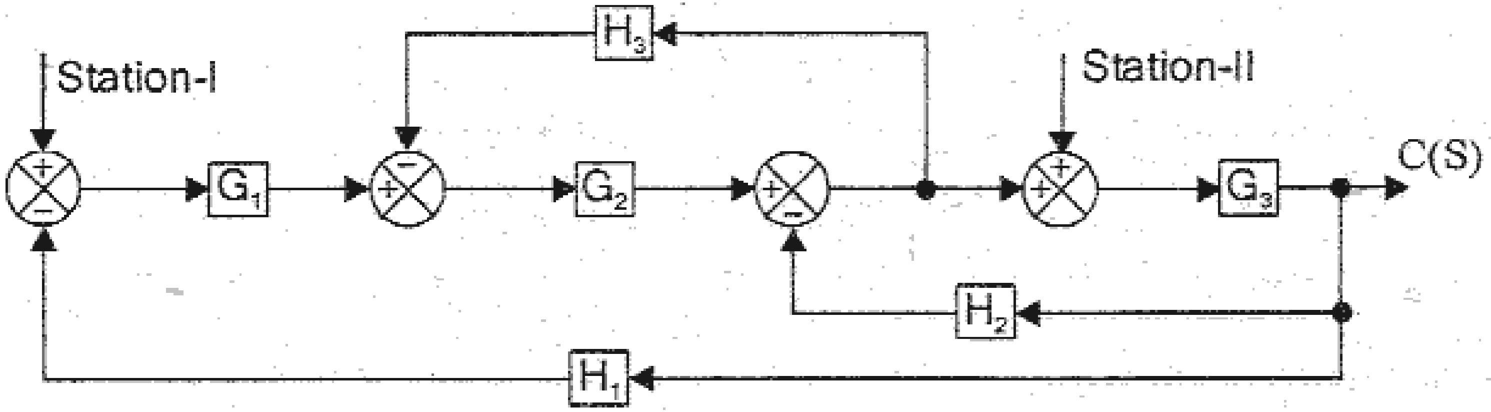


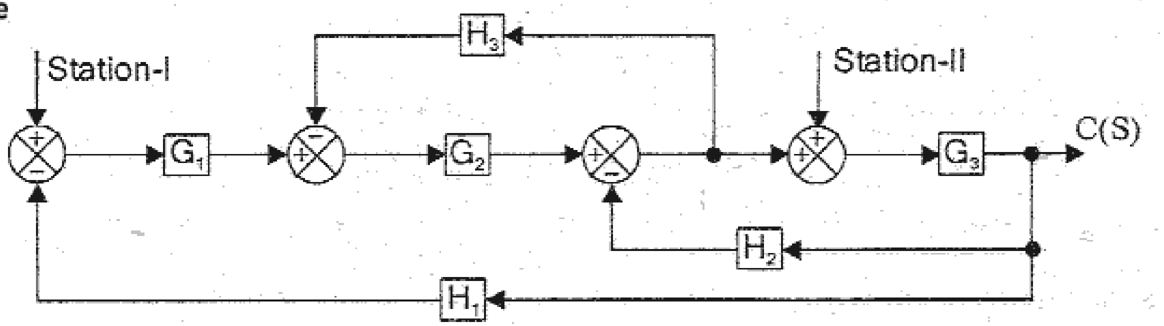
$$\frac{\dfrac{G_2(1+G_3)}{1+G_2-G_2H_1}}{1-\dfrac{G_2\,(1+G_3)\,HH_1}{1+G_2-G_2H_1}}$$



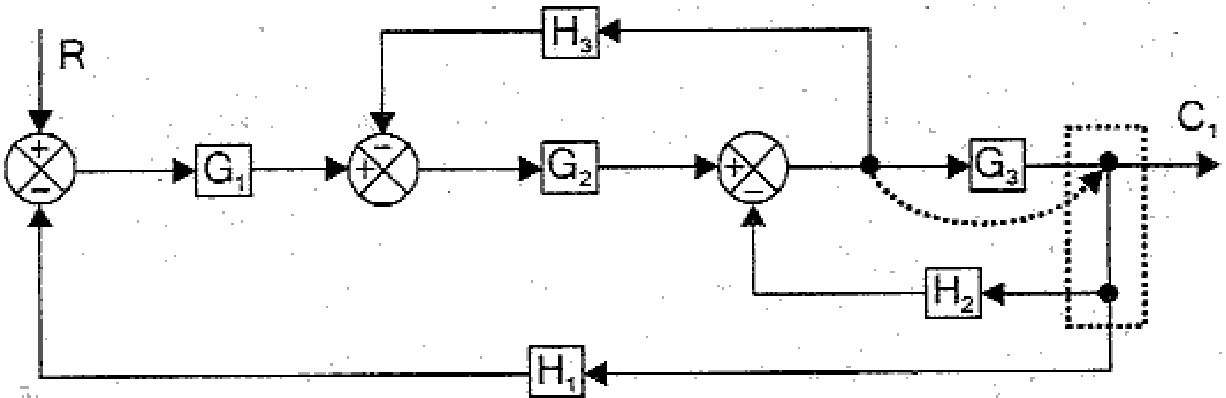
$$\frac{C(s)}{R(s)} = \frac{G_1G_2 + G_1G_2G_3}{1+G_2-G_2H_1-G_2HH_1-G_2G_3HH_1}$$

9.For the system represented by the block diagram shown in figure and Evaluate the closed loop transfer function when the input R is (i) at Station-I (ii) at station-II

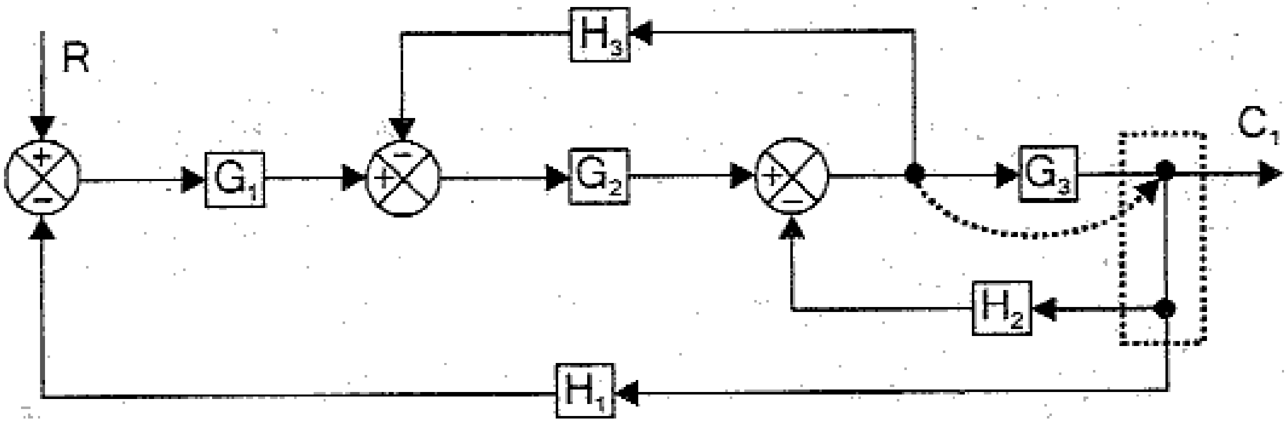


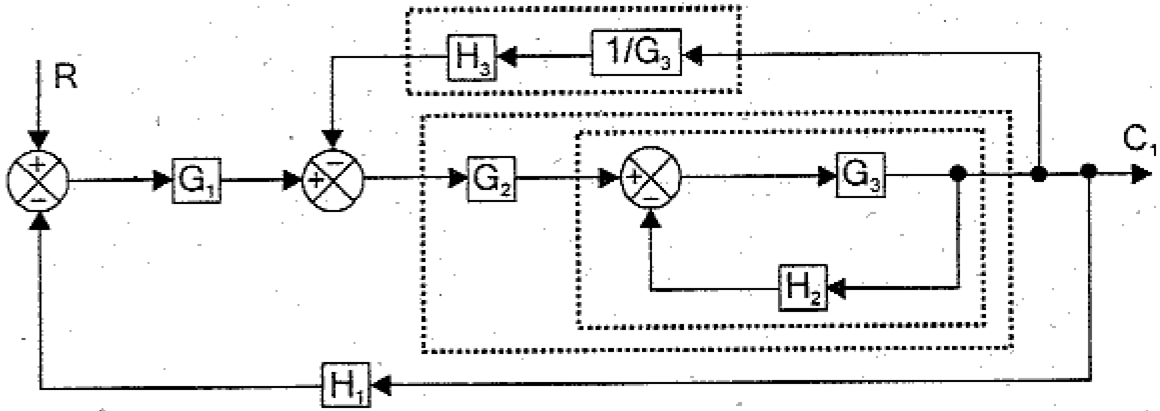


Consider the input R is at station-I and so the input at station-II is made zero. Let the output be C1. Since there is no input at station-II that summing point can be removed and resulting block diagram is shown in below fig.

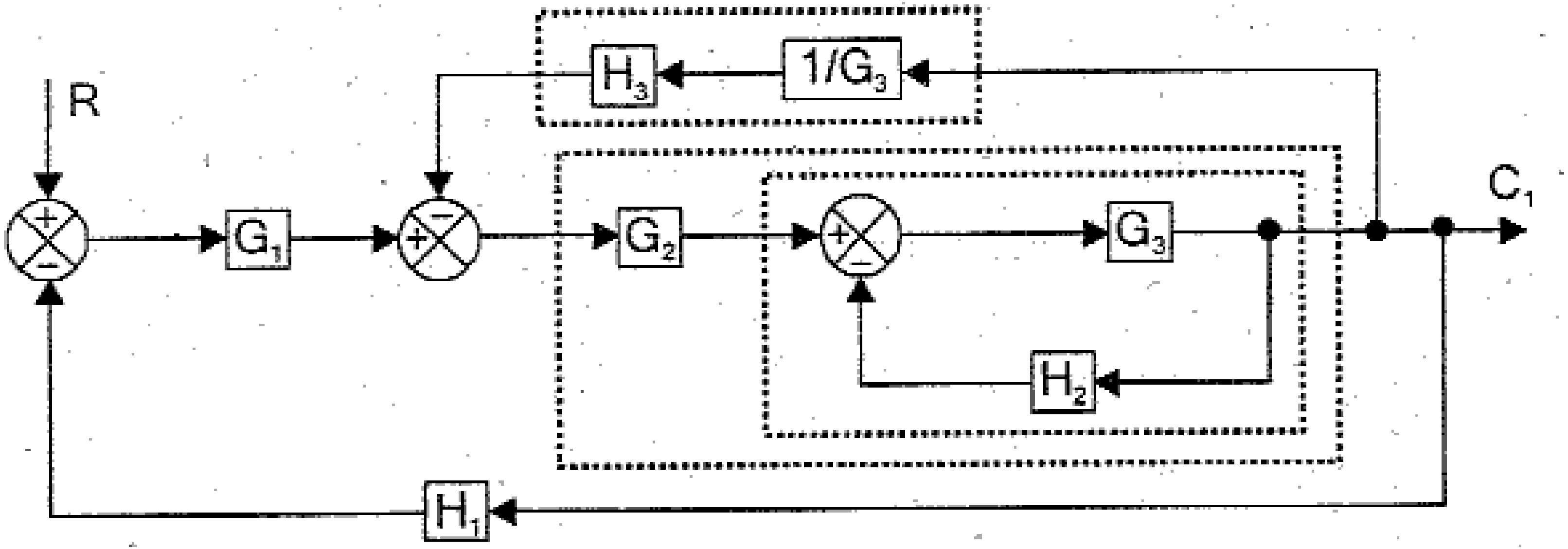


Step 1: Shift the take off point of feedback H3 beyond G3 and rearrange the branch points

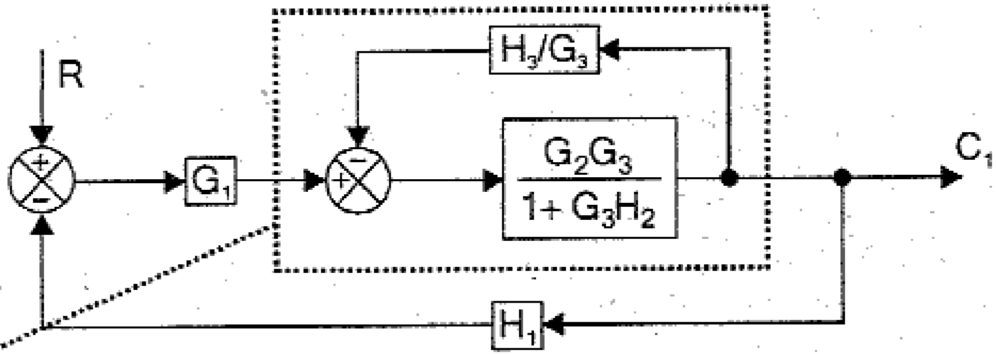




Step 2: Eliminating the feedback H2 and combining blocks in cascade

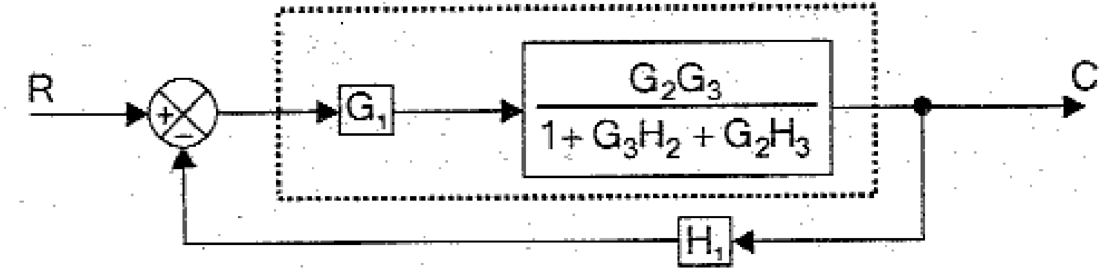


Step 3: Eliminating the feedback path

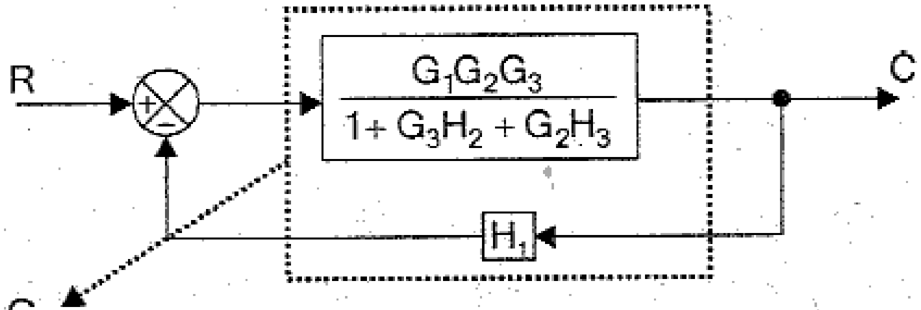


$$\frac{\frac{G_2G_3}{1+G_3H_2}}{1+\frac{G_2G_3}{1+G_3H_2}\times\frac{H_3}{G_3}} = \frac{\frac{G_2G_3}{1+G_3H_2}}{1+\frac{G_2H_3}{1+G_3H_2}} = \frac{G_2G_3}{1+G_3H_2+G_2H_3}$$

Step 4: Combining the blocks in cascade

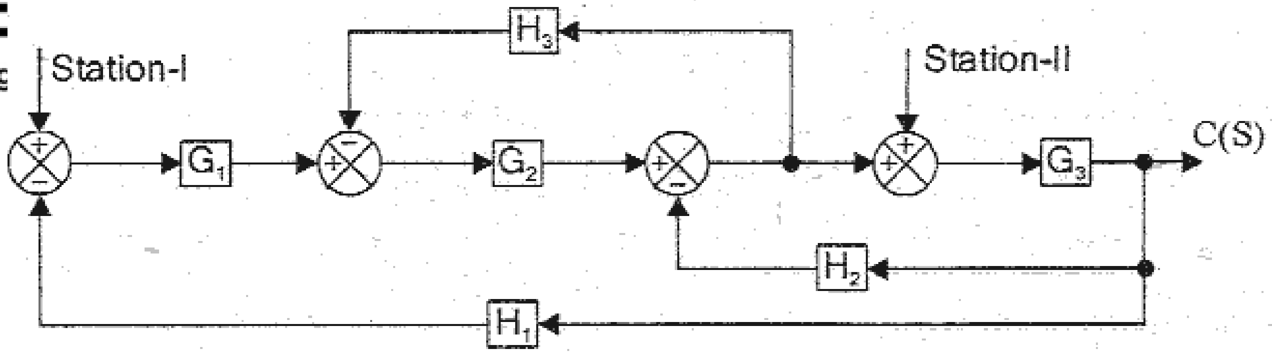


Step 5: Eliminating feedback path H1

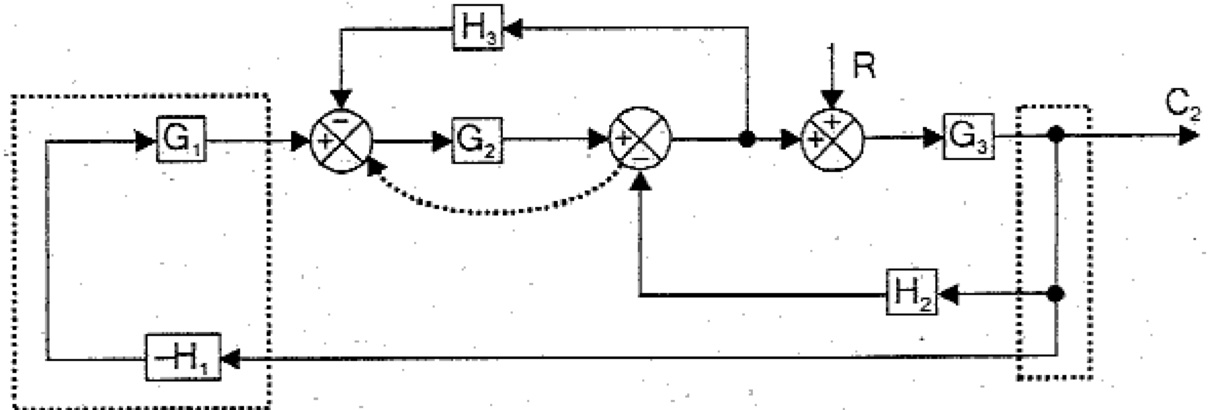


$$\frac{\dfrac{G_1G_2G_3}{1+G_3H_2+G_2H_3}}{1+\dfrac{G_1G_2G_3}{1+G_3H_2+G_2H_3}\times H_1} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

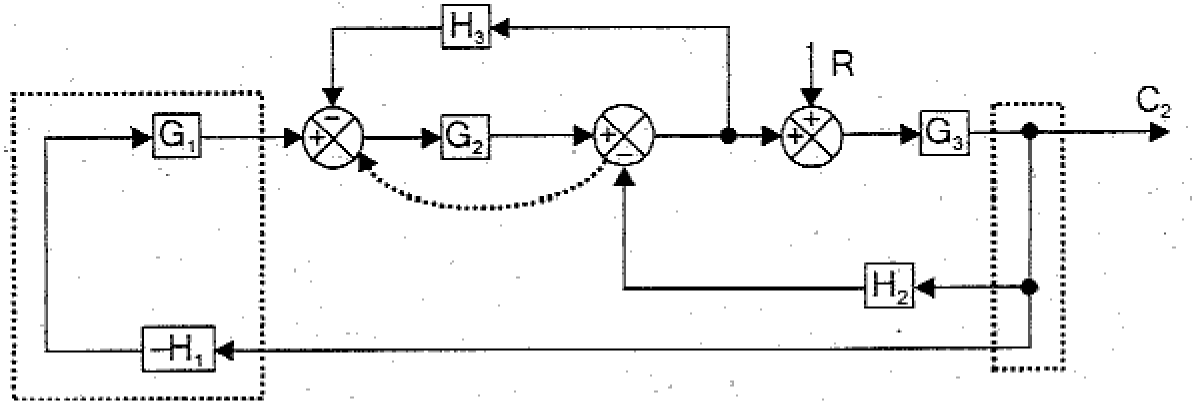
$$\therefore \frac{C_1(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

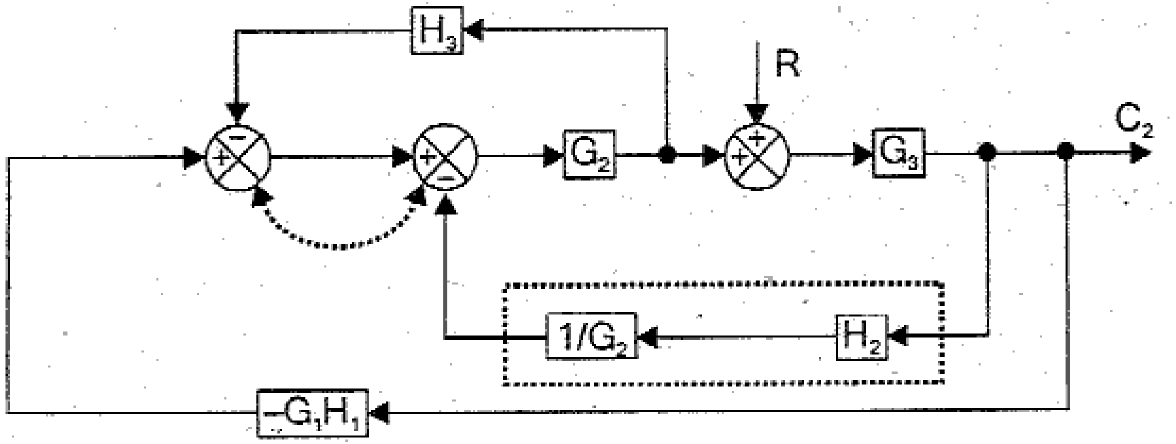


Consider the input R at station-II, the input at station-I is made zero. Let output be C2. Since there is no input in station-1 that corresponding summing point can be removed and a negative sign can be attached to the feedback path gain H1. The resulting block diagram is shown in fig 3.

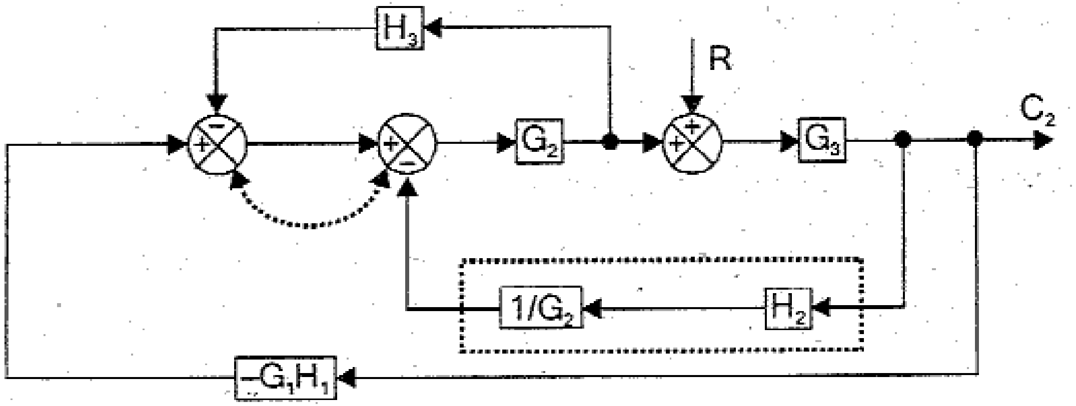


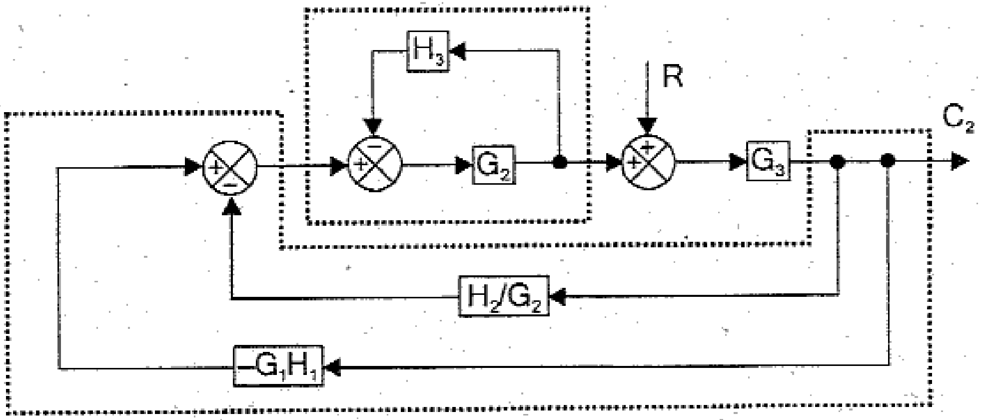
Step 1: Combining the blocks in cascade, shifting the summing point of H2 before G2 and rearranging the branch point



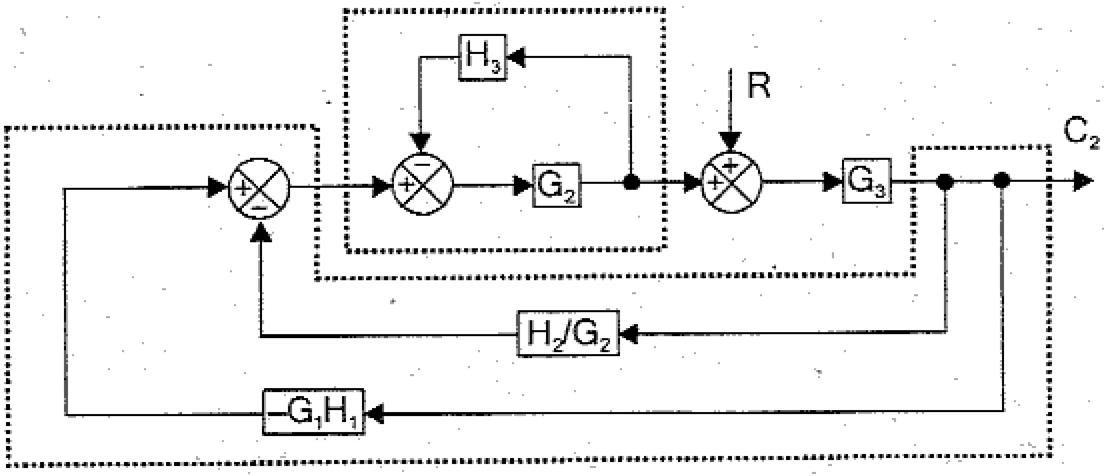


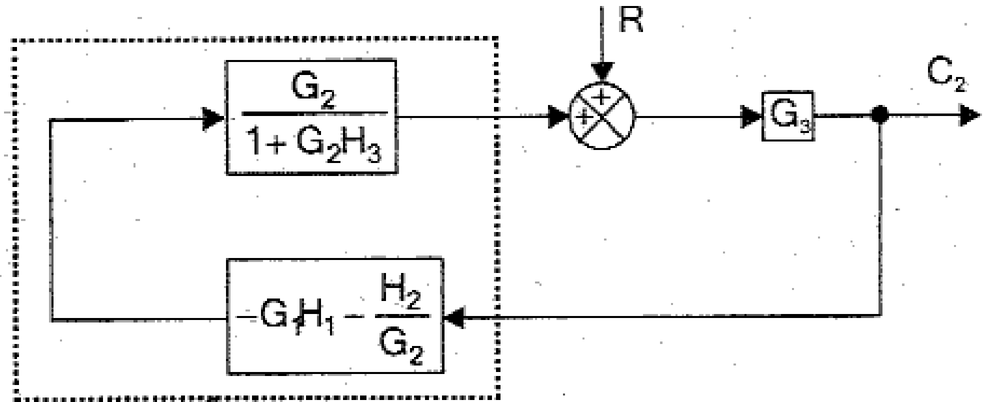
Step 2: Interchanging summing points and combining the blocks in cascade.



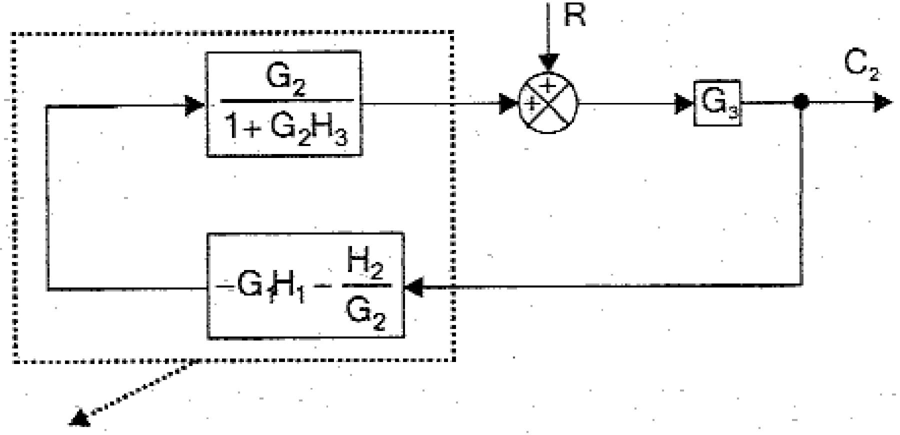


Step 3: Combining parallel blocks and eliminating feedback path



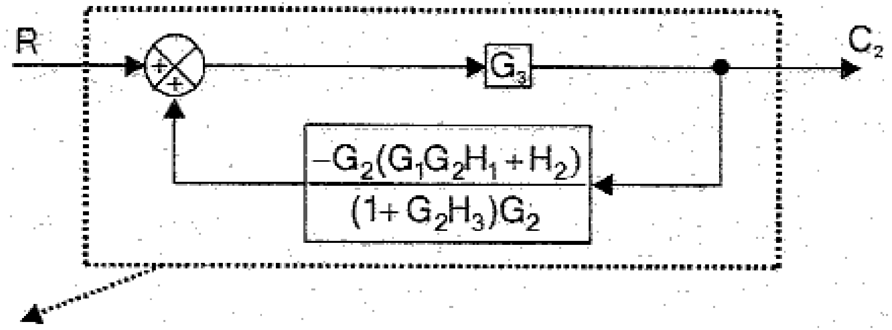


Step 4: Combining the blocks in cascade



$$\left(\frac{G_2}{1+G_2H_3}\right)\times\left(-G_1H_1-\frac{H_2}{G_2}\right)=\left(\frac{G_2}{1+G_2H_3}\right)\times\left(\frac{-G_1H_1G_2-H_2}{G_2}\right)=\frac{-G_2(G_1G_2H_1+H_2)}{(1+G_2H_3)G_2}$$

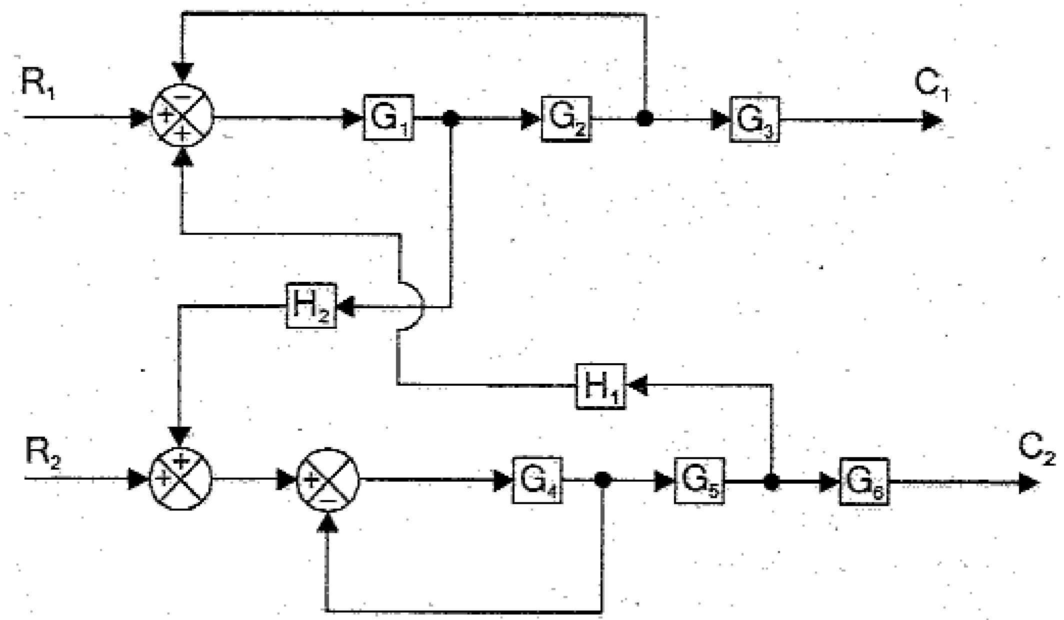
Step 5: Eliminating the feedback path



$$\frac{G_3}{1-\left(\frac{-(G_1G_2H_1+H_2)}{1+G_2H_3}\right)G_3} = \frac{G_3}{\frac{1+G_2H_3+G_3(G_1G_2H_1+H_2)}{1+G_2H_3}} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3(G_1G_2H_1+H_2)}$$

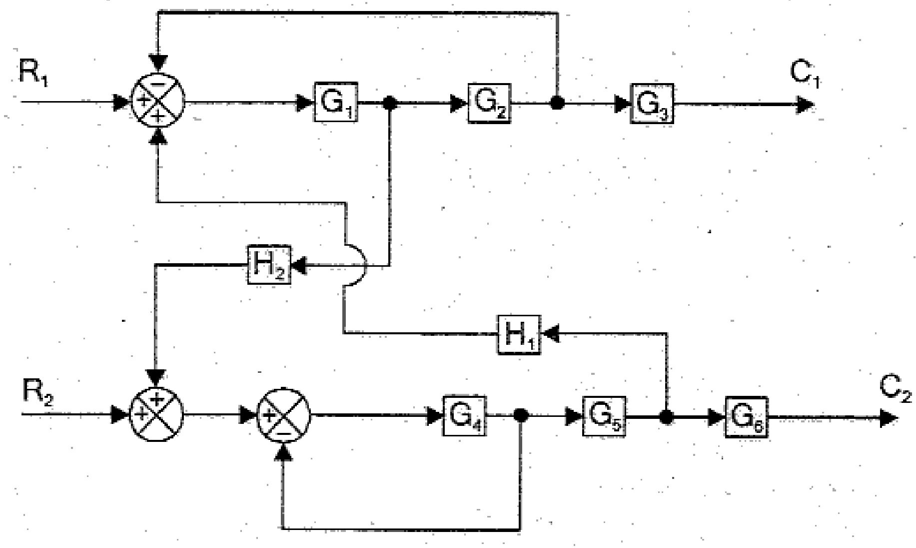
$$\therefore \frac{C_2}{R} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3(G_1G_2H_1+H_2)}$$

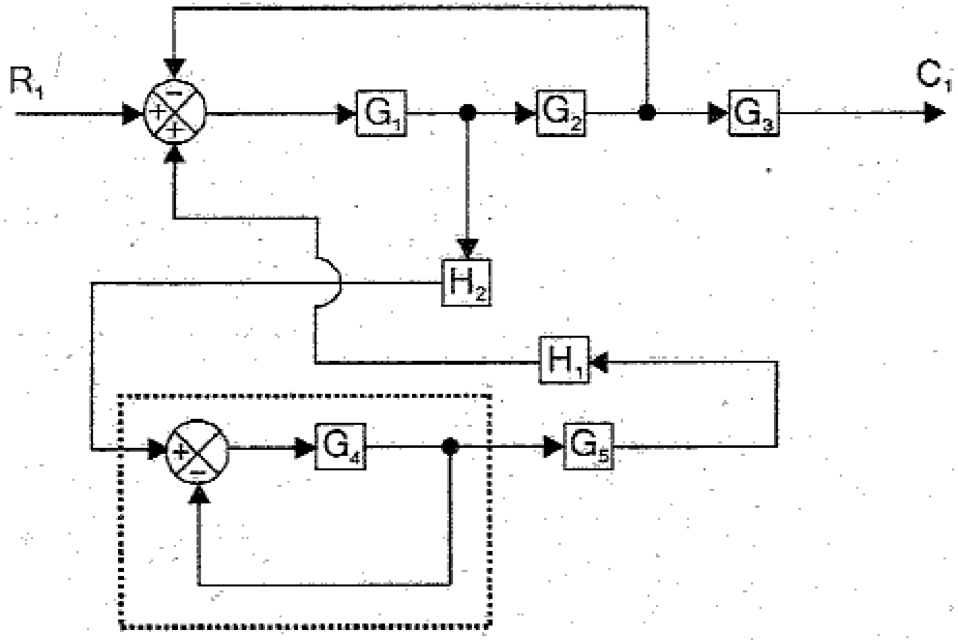
10.For the system represented by the block diagram shown in figure. Determine C1/R1 and C2/R1



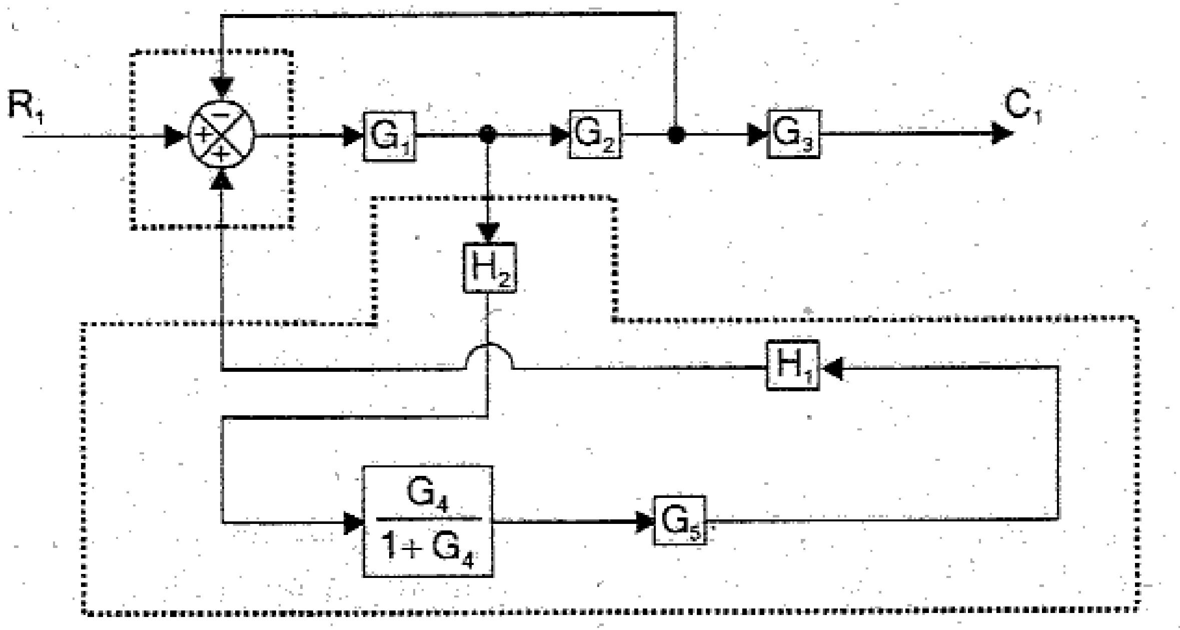
Case1 to Find C1/R1

Set R2=0 and consider only one output C1.Hence we can remove the summing point which adds R2 and need not consider G6, since G6 is on the open path. The resulting block diagram is shown

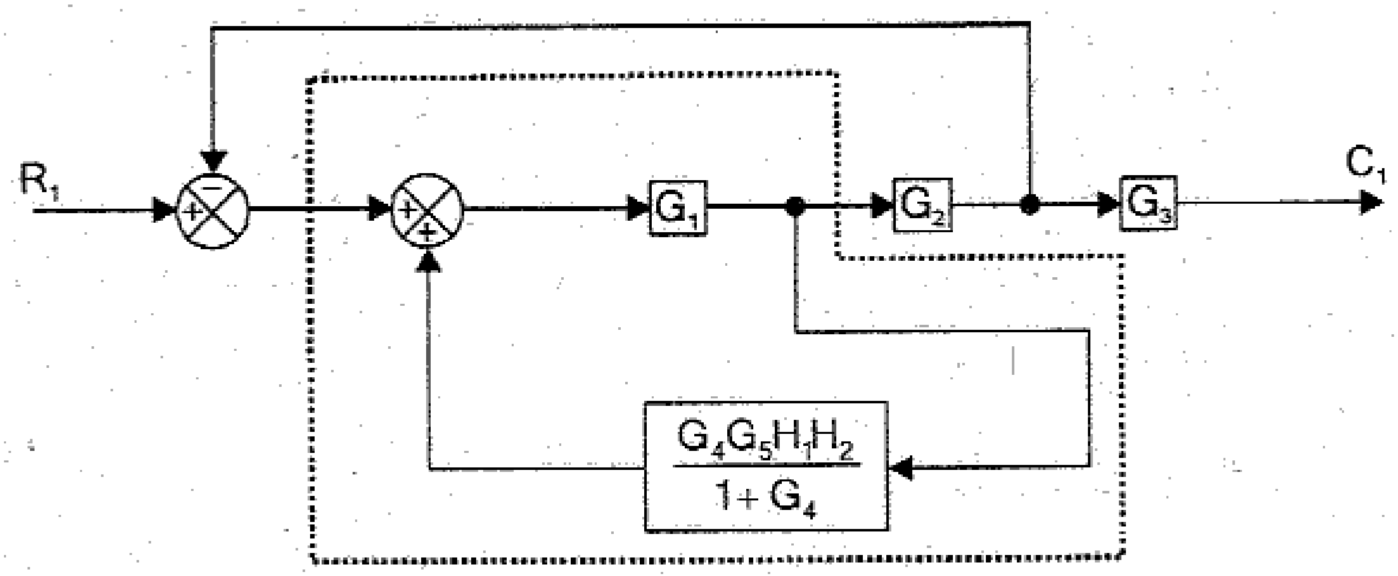




Combine the blocks in cascade & Splitting the summing point

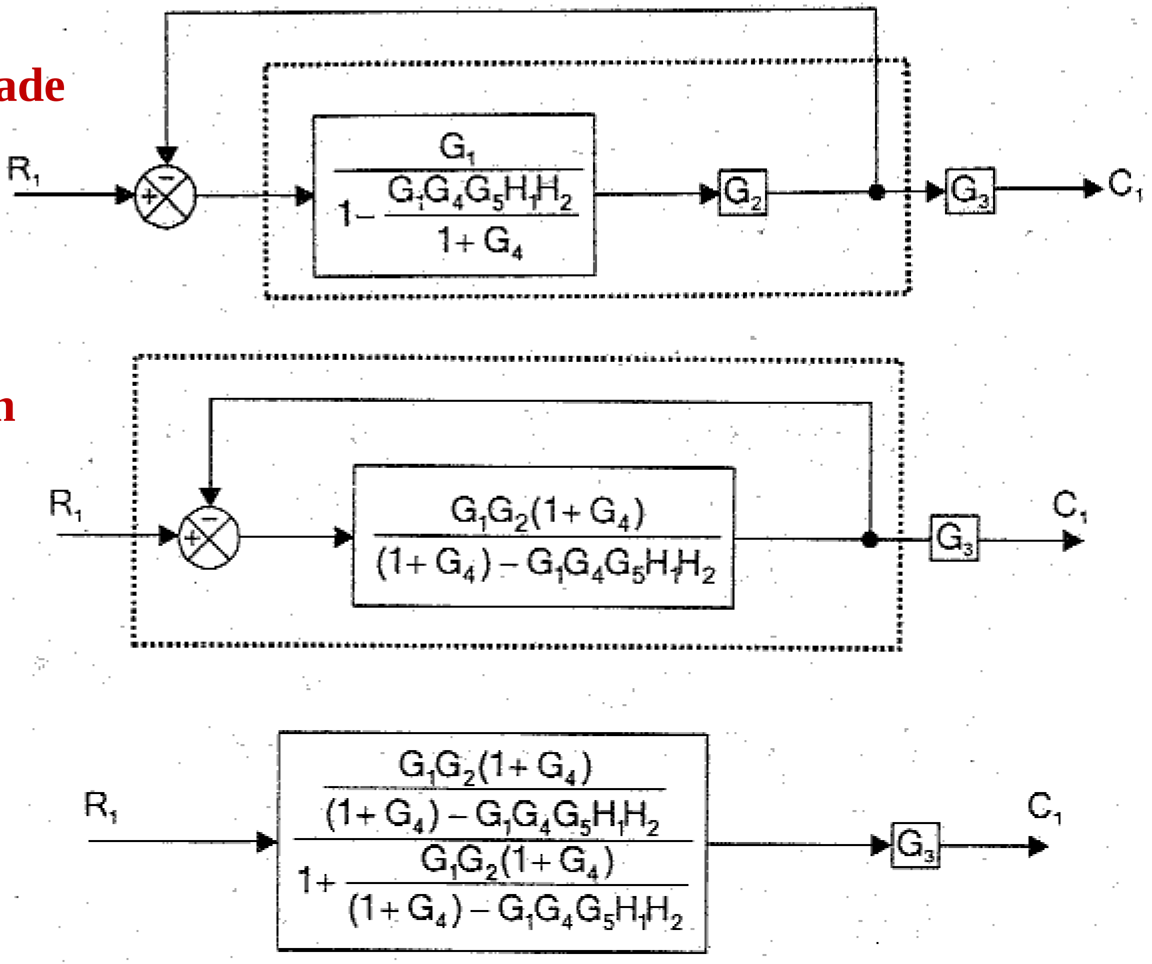


Eliminating feedback path

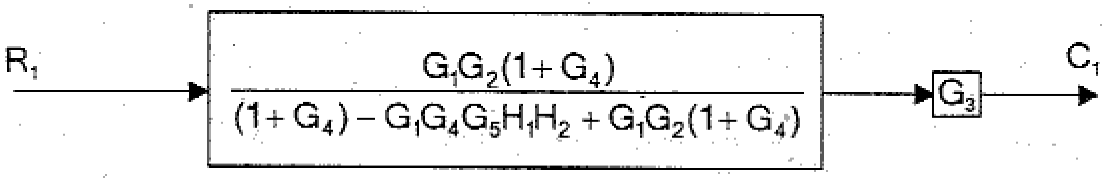


Combining blocks in cascade

Eliminating feedback path



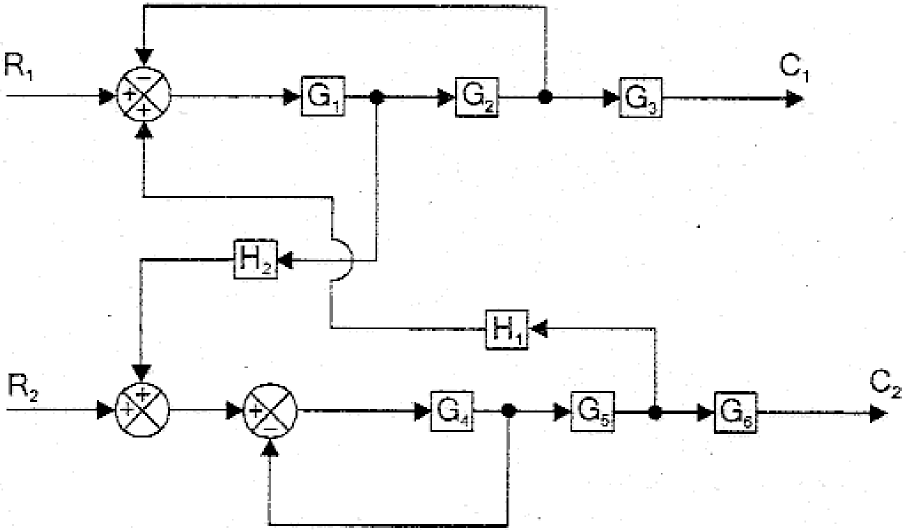
Combining blocks in cascade

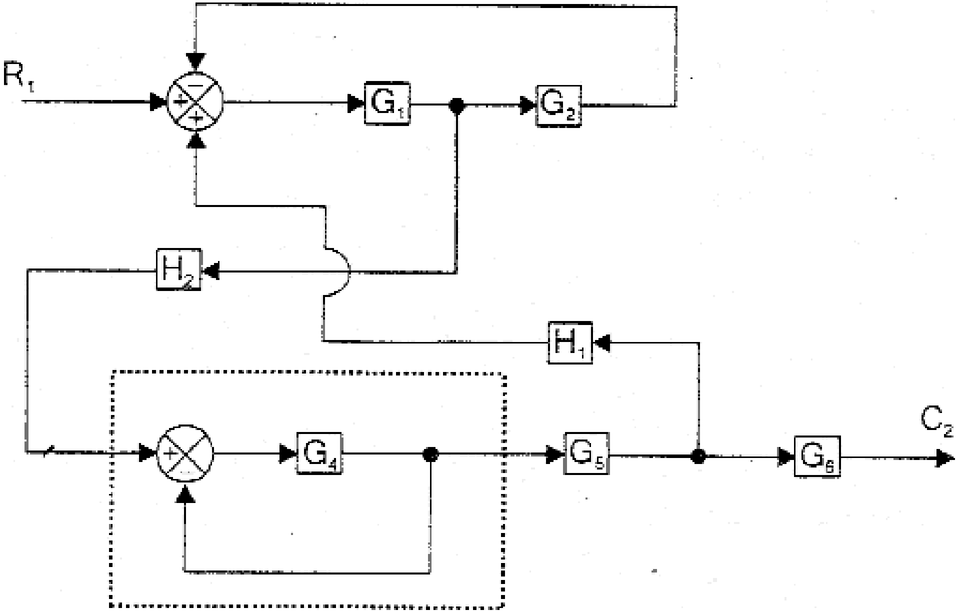


$$\frac{C_1}{R_1}=\frac{G_1G_2G_3(1+G_4)}{(1+G_1G_2)\,(1+G_4)-G_1G_4G_5H_1H_2}$$

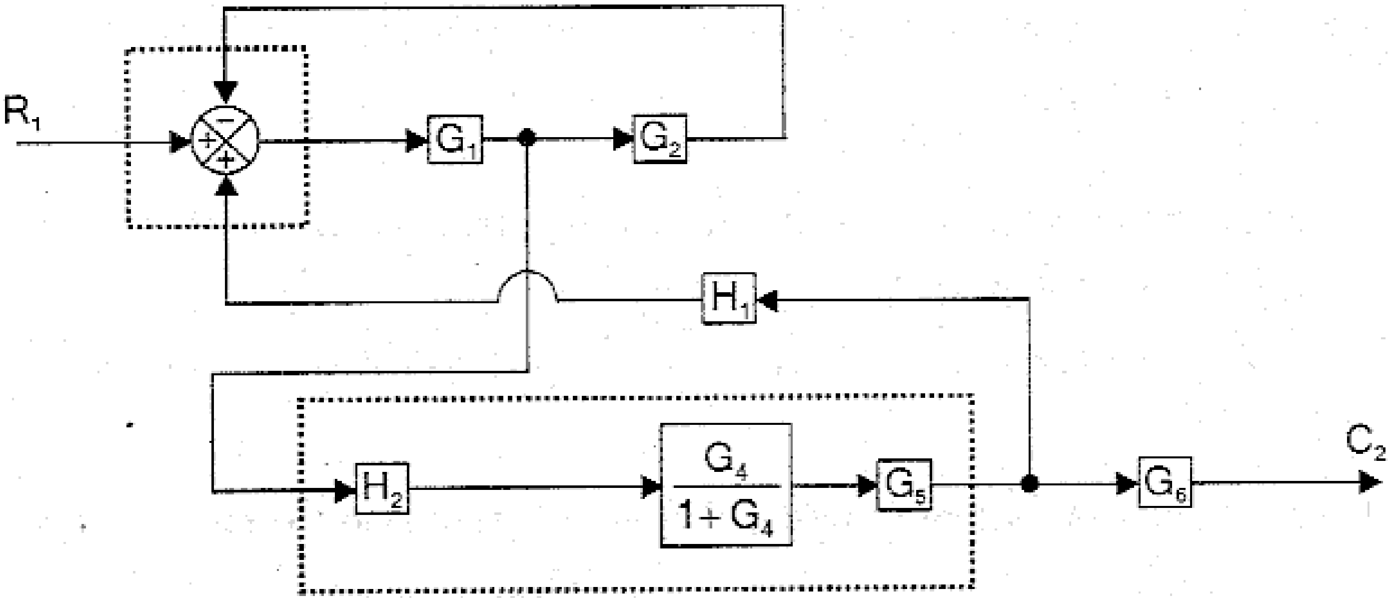
Case2 to Find C2/R1

Set R2=0 and consider only one output C2.Hence we can remove the summing point which adds R2 and need not consider G3, since G3 is on the open path. The resulting block diagram is shown

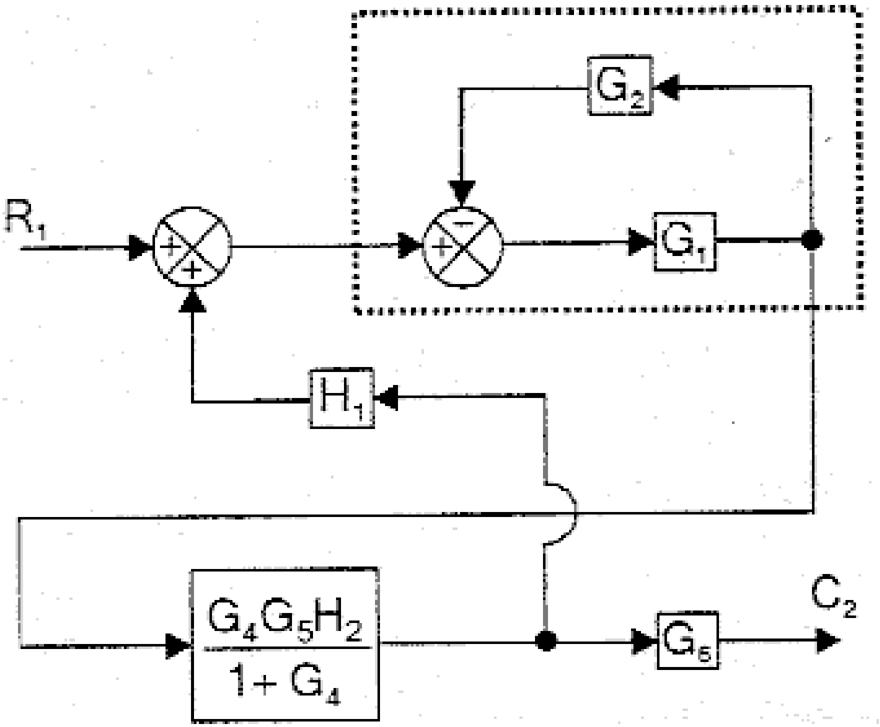




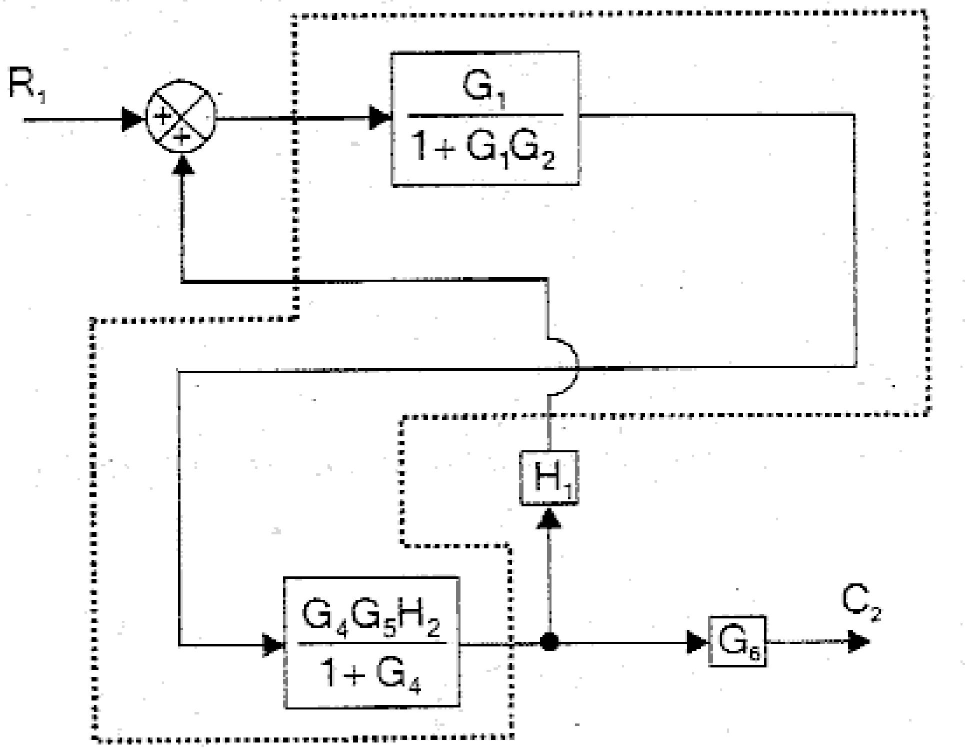
Combine the blocks in cascade & Splitting the summing point



Eliminating feedback path



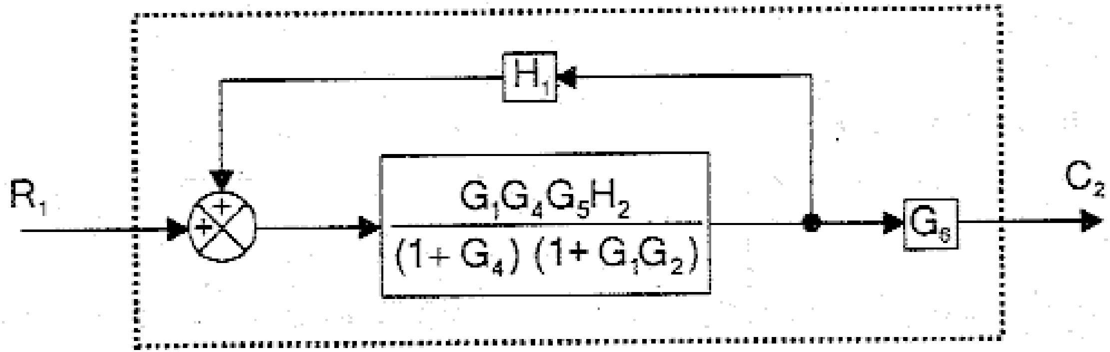
Combining blocks in cascade

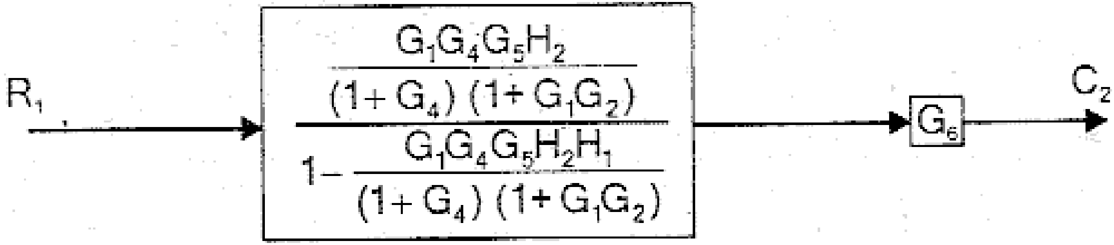


ATME College of Engineering

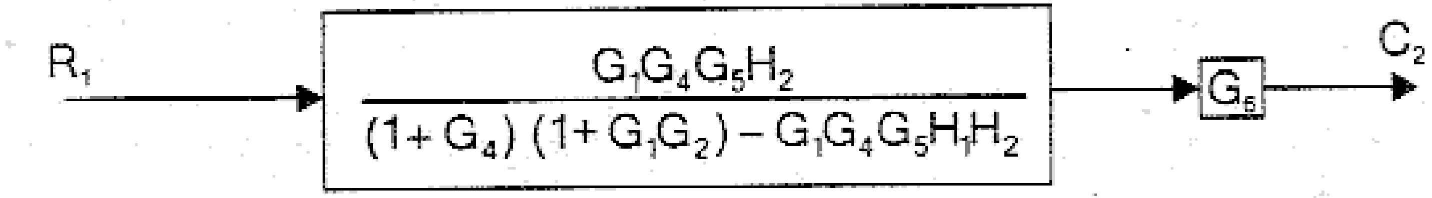
Department of EEE Emitting Elite Energy

Eliminating feedback path





Combining blocks in cascade



$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{(1+G_4)\,(1+G_1G_2) - G_1G_4G_5H_1H_2}$$

BLOCK DIAGRAM

Advantages of Block Diagram

The various advantages of block diagram representation are,

1) Very simple to construct the block diagram for complicated systems.
2) The function of individual element can be visualised from block diagram.
3) Individual as well as overall performance of the system can be studied by using transfer functions shown in the block diagram.
4) Overall closed loop T.F. can be easily calculated by using block diagram reduction rules.

Disadvantages

The various disadvantages of block diagram representation are,

1) Block diagram does not include any information about the physical construction of the system.
2) Source of energy is generally not shown in the block diagram. So number of different block diagrams can be drawn depending upon the point of view of analysis. So block diagram for given system is not unique.

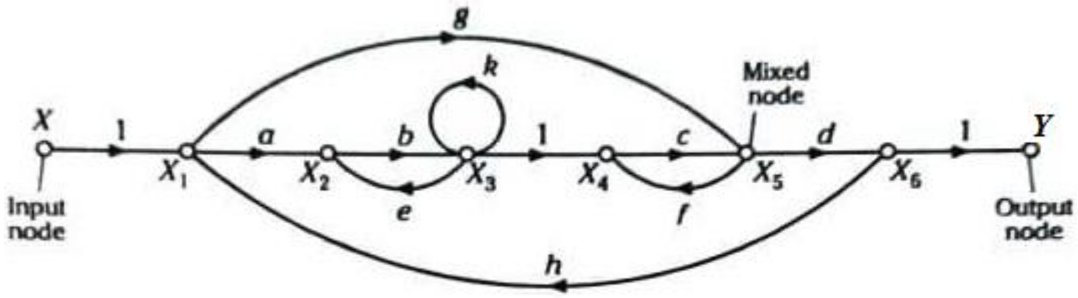
SIGNAL FLOW GRAPH

The signal flow graph is used to represent the control system graphically and it was developed by S.J. Mason.

A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. By taking Laplace transform, the time domain differential equations governing a control system can be transferred to a set of algebraic equations in s-domain. The signal flow graph of the system can be constructed using these equations.

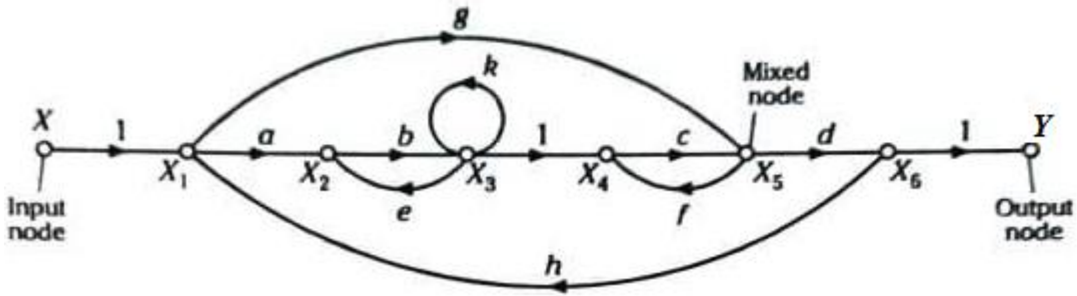
The signal flow graph approach and the block diagram approach yield the same information. The advantage in signal flow graph method is that, using Mason's gain formula the overall gain of the system can be computed easily. This method is simpler than the tedious block diagram reduction techniques.

The signal flow graph depicts the flow of signals from one point of a system to another and gives the relationships among the signals.



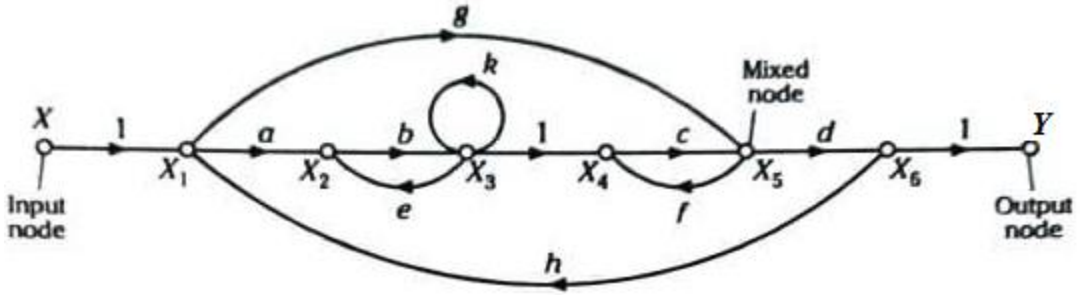
A signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable and each branch connected between two nodes acts as a signal multiplier. Each branch has a gain or transmittance. When the signal pass through a branch, it gets multiplied by the gain of the branch.

In a signal flow graph, the signal flows only one direction. The direction of signal flow is indicated by an arrow placed on the branch and the gain (multiplication factor) is indicated along the branch.



DEFINITIONS

Node: A node is a point representing a variable or signal.

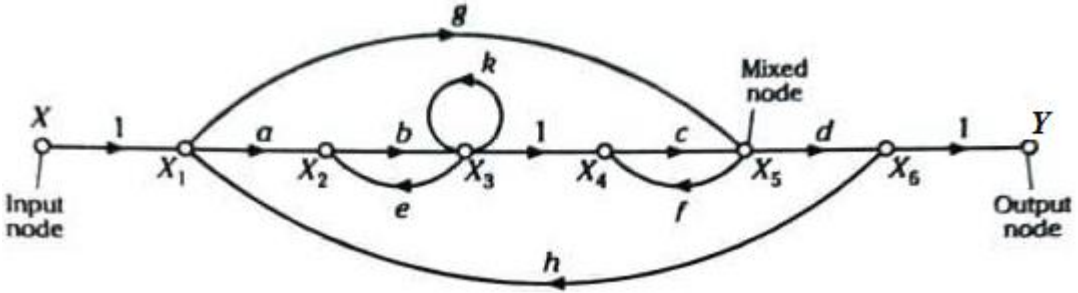


Branch: A branch is directed line segment joining two nodes. The arrow on the branch indicates the direction of signal flow and the gain of a branch is the transmittance

Transmittance : The gain acquired by the signal when it travels from one node to another called transmittance. The transmittance can be real or complex.

DEFINITIONS

Input node (Source) : It is a node that has only outgoing branches. Node Representing Variable X is Input Node

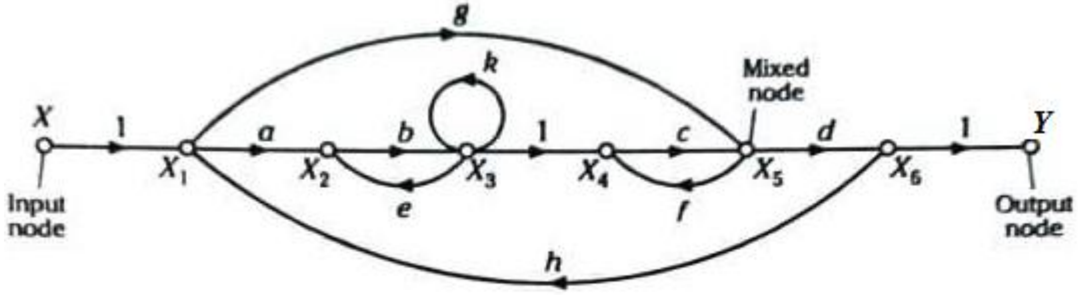


Output node (Sink) : It is a node that has only incoming branches, Node Representing Variable Y is output Node

Mixed node: It is a node that has both incoming and outgoing branches. for example X5 Node is a Mixed Node

DEFINITIONS

Path: Path is a traversal of connected branches in the direction of the branch arrows. The path Should not cross a node more than once.



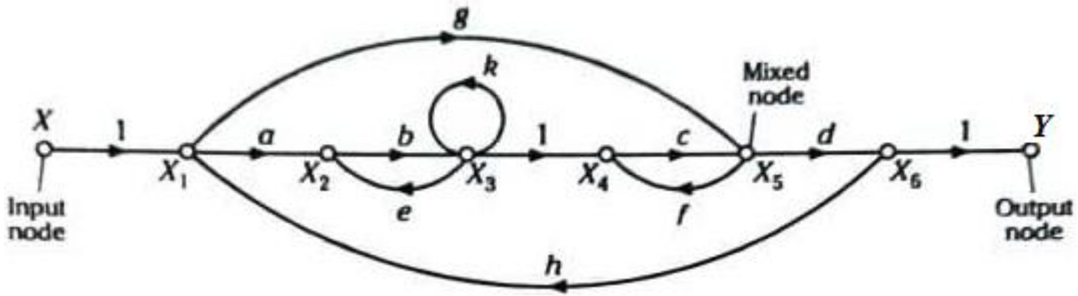
Open path : A open path starts at a node and ends at another node.

Closed path: Closed path starts and ends at same node.

Forward path : It is a path from an input node to an output node that does not cross any node more than once.

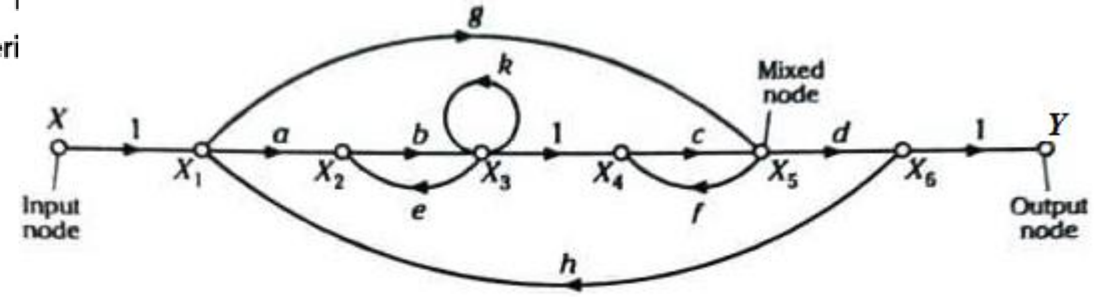
Forward path gain It is the product of the branch transmittances (gains) of a forward path.

There are two forward paths in the given example and the forward path gains are P1 = abcd, P2 = gd.



Individual loop : It is a closed path starting from a node and after passing through a certain part of a graph arrives at same node without crossing any node more than once.

Self-loop : The loop starting at node X3 and ending at same node represents self-loop. L5 is a self-loop.



Loop gain: It is the product of the branch transmittances (gains) of a loop. There are 5 Loops in given example and loop gain are L1= be, L2 = cf, L3 = abcdh, L4 = gdh, L5= k

Non-touching Loops: If the loops does not have a common node then they are said to be nontouching loops. L1 and L2 are Non touching loops and L1 and L4 are Non touching loops, L2 & L5, L4 & L5 are Non touching loops

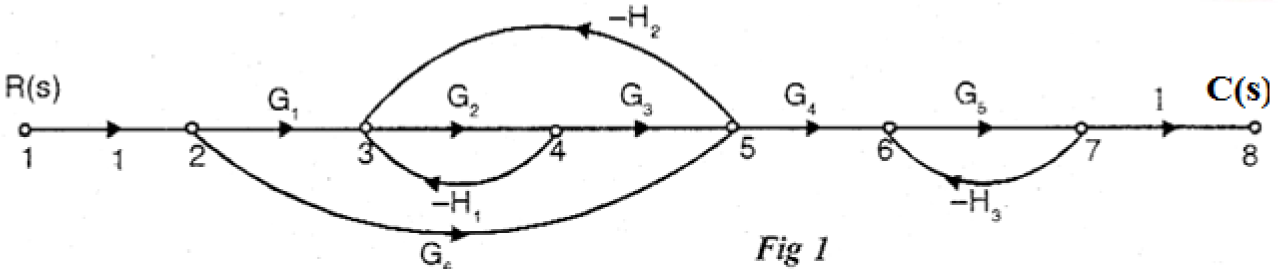


Fig 1

Forward Path Gains

Forward Path-1, Gain of forward path-1, P1 =G1G2G3G4G5

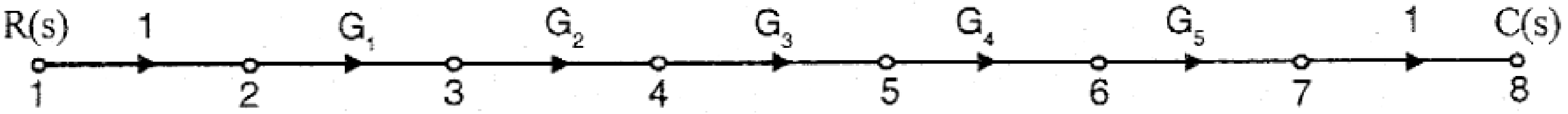


Fig 2 : Forward path-1.

Forward Path-2, Gain of forward path-2, P2 = G4G5G6

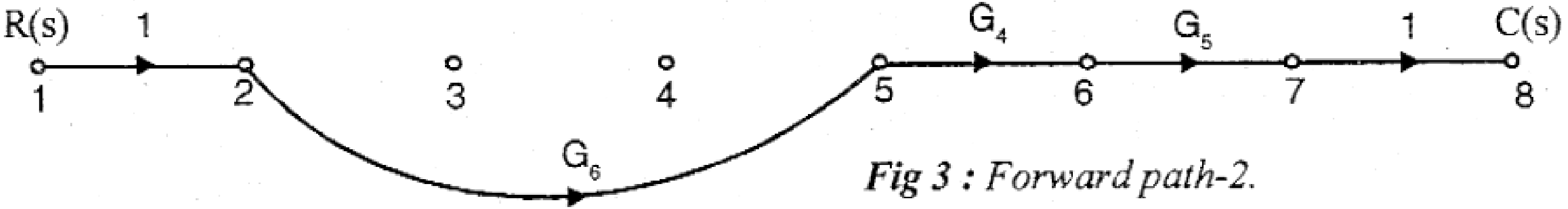


Fig 3 : Forward path-2.

Individuals Loop

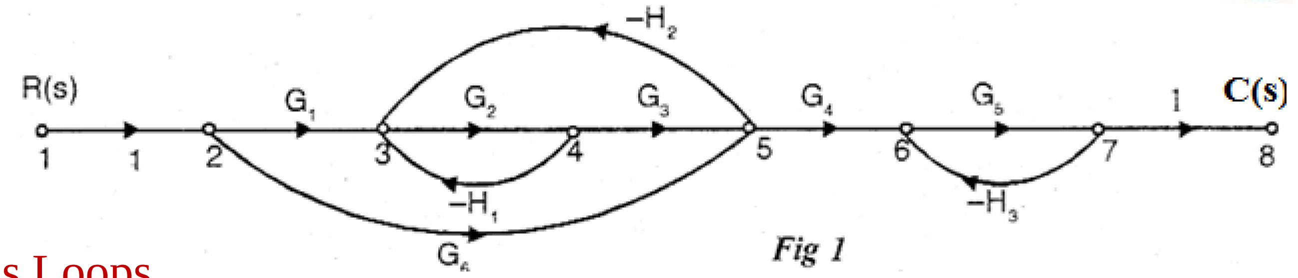


Fig 1

There are 3 Individuals Loops

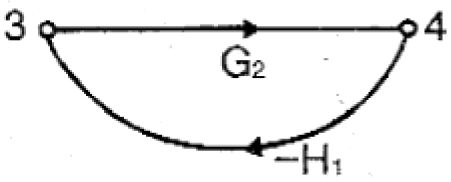


***Fig 4 :** Loop-1.*

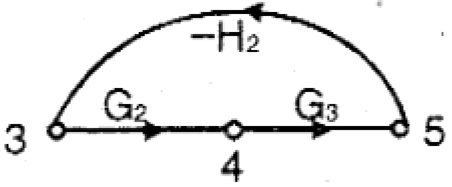


***Fig 5 :** Loop-2.*

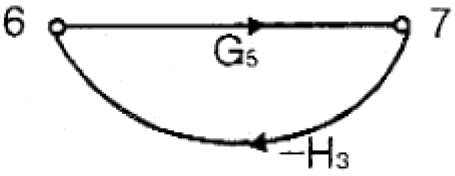


***Fig 6 :** Loop-3.*

Loop Gain of Individuals Loop-1, L1 = -G2H1

Loop Gain of Individuals Loop-2, L2 = -G2G3H2

Loop Gain of Individuals Loop-3, L3 = -G5H3

Gain Products of Two Non-Individuals Loop

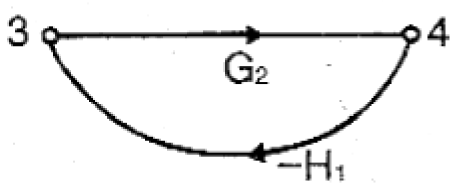


Fig 4 : Loop-1.

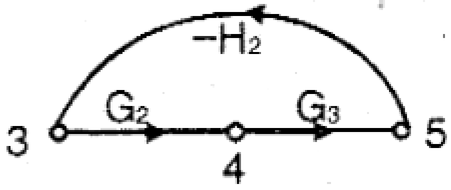


Fig 5 : Loop-2.

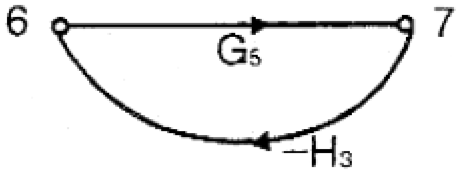


Fig 6 : Loop-3.

There are Two combinations of Two Non-touching loops

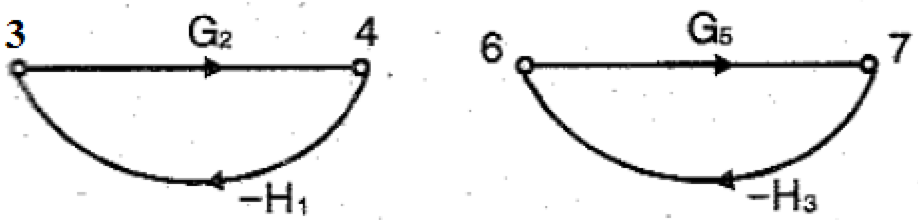


Fig.7 First combinations of 2 Non-touching loops

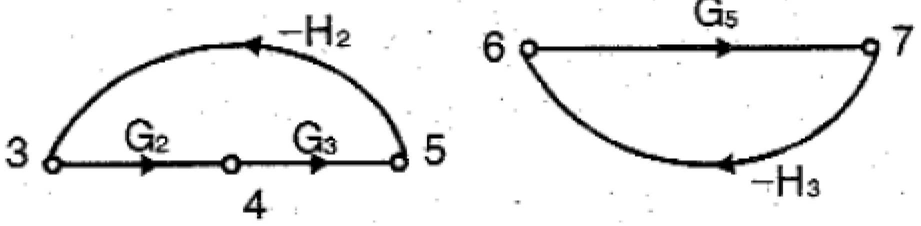


Fig.8 Second combinations of 2 Non-touching loops

Gain Product of 1st combinations of 2 Non-touching loops L13 = L1*L3 =(-G2H1)*(-G5H3) = G2G5H1H3

Gain Product of 2nd combinations of 2 Non-touching loops L23 = L2*L3 =(-G2G3H2)*(-G5H3) = G2G3G5H2H3

The basic properties of signal flow graph

(i) Signal flow graph is applicable to linear systems only.

(ii) A node in the signal flow graph represents the variable or signal.

(iii) A node adds the signals of all incoming branches and transmits the sum to all outgoing branches.

(iv) A mixed node which has both incoming and outgoing signals can be treated as an output node by adding an outgoing branch of unity transmittance

(v) A branch indicates functional dependence of one signal on the other

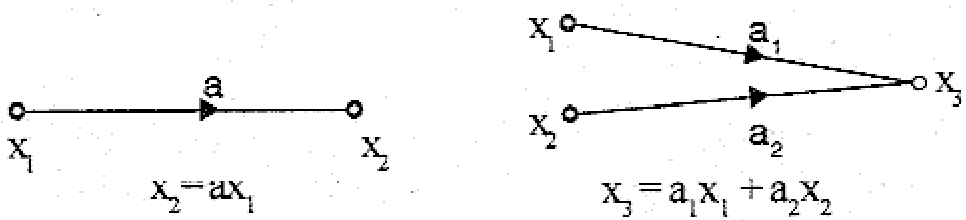
(vi) The signals travel along branches only in the marked direction and when it travels it gets multiplied by the gain or transmittance of the branch.

(vii) The signal flow graph of system is not unique. By rearranging the system equations different types of signal flow graphs can be drawn for a given system.

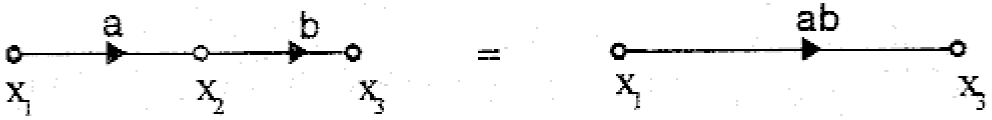
SIGNAL FLOW GRAPH ALGEBRA

Signal flow graph for a system can be reduced to obtain the transfer function of the system using the following rules. The guideline in developing the rules for signal flow graph algebra is that the signal at a node is given by sum of all incoming signals.

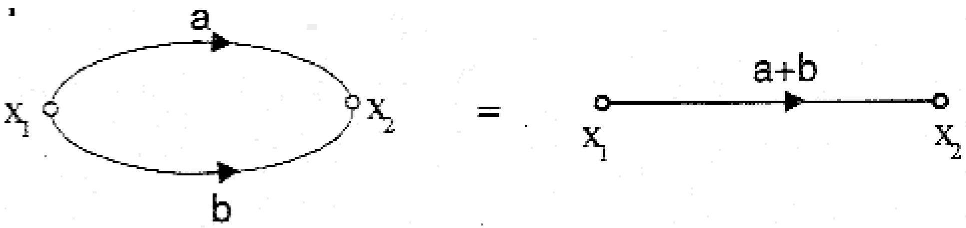
Rule 1 : Incoming signal to a node through a branch is given by the product of a signal at previous node and the gain of the branch.



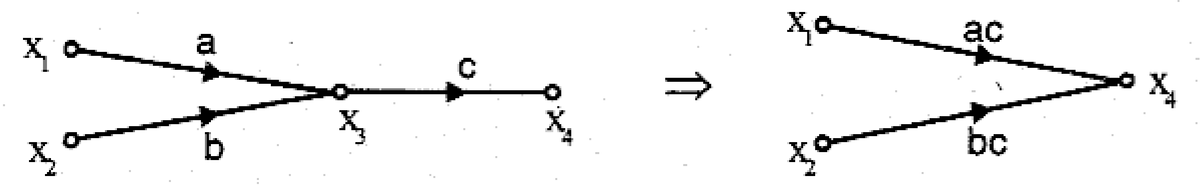
Rule 2 : Cascaded branches can be combined to give a single branch whose transmittance is equal to the product of individual branch transmittance.



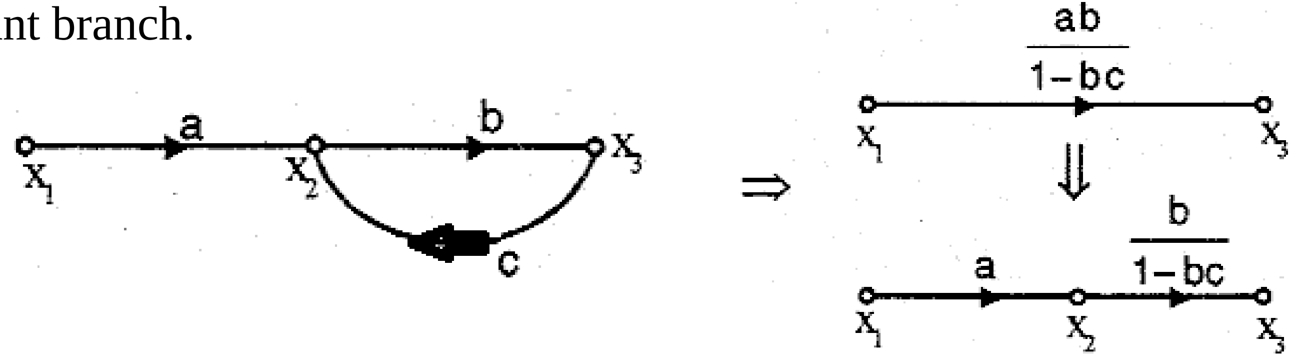
Rule 3 : Parallel branches may be represented by single branch whose transmittance is the sum of individual branch transmittances.



Rule 4: A mixed node can be eliminated by multiplying the transmittance of outgoing branch (from the mixed node) to the transmittance of all incoming branches to the mixed node



Rule 5 : A loop may be eliminated by writing equations at the input and output node and rearranging the equations to find the ratio of output to input. This ratio gives the gain resultant branch.



$$x_2 = ax_1 + cx_3 \;;\quad x_3 = bx_2$$

Put, $x_2 = ax_1 + cx_3$ in the equation for x_3.

$$\therefore x_3 = b\,(ax_1 + cx_3) \;\Rightarrow\; x_3 = abx_1 + bcx_3 \;\Rightarrow\; x_3 - bc\,x_3 = ab\,x_1 \;\Rightarrow\; x_3(1 - bc) = ab\,x_1$$

$$\therefore \frac{x_3}{x_1} = \frac{ab}{1 - bc}$$

S.J.Mason bas developed a simple procedure to determine the transfer function of the system represented as a signal flow graph. He has developed a formula called by his name **Mason's gain formula** which can be directly used to find the transfer function of the system.

The Mason's gain formula is used to determine the transfer function of the system from the signal flow graph of the system

Let, R(s) = Input to the system, C(s) = Output of the system C(s)

Transfer function of the system, T(s) = C(s)/R(s)

Mason's gain formula states the overall gain of the system (transfer function] as follows,

Overall gain, $$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$$

T = $T(s)$ = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

Determinant of SFG Δ = 1 – (Sum of individual loop gains)

+ (Sum of gain products of all possible combinations of two non - touching loops)

– (Sum of gain products of all possible combinations of three non - touching loops)

+ ....

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

$$\text{Overall T.F.} = \frac{\Sigma\, T_K\, \Delta_K}{\Delta}$$

where K = Number of forward paths

T_K = Gain of K^{th} forward path

Δ = System determinant to be calculated as :

Δ = 1–[Σ all individual feedback loop gains [including self loops]] + [Σ Gain × Gain product of all possible combinations of two non-touching loops] – [Σ Gain × Gain × Gain product of combinations of three non touching loops] +....

Δ_K = Value of above Δ by eliminating all loop gains and associated products which are touching to the K^{th} forward path.

CONSTRUCTING SIGNAL FLOW GRAPH FOR CONTROL SYSTEMS

A control system can be represented diagrammatically by signal flow graph. The differential equations governing the system are used to construct the signal flow graph

1. Take Laplace transform of the differential equations governing the system in order to convert them to algebraic equations in s-domain.

2. The constants and variables of the s-domain equations are identified.

3.From the working knowledge of the system, the variables are identified as input, output and intermediate variables.

4. For each variable a node is assigned in signal flow graph and constants are assigned as the gain or transmittance of the branches connecting the nodes.

5. For each equation a signal flow graph is drawn and then they are interconnected to give overall signal flow graph of the system.

PROCEDURE FOR CONVERTING BLOCK DIAGRAM TO SIGNAL FLOW GRAPH

1. Assume nodes at input, output, at every summing point, at every branch point and in between cascaded blocks.

2. Draw the nodes separately as small circles and number the circles in the order 1, 2, 3, 4, etc.

3. From the block diagram find the gain between each node in the main forward path and connect all the corresponding circles by straight line and mark the gain between the nodes.

4. Draw the feed forward paths between various nodes and mark the gain of feed forward path along with sign

5. Draw the feedback paths between various nodes and mark the gain of feedback paths along with sign.

Find the overall transfer function of the system whose signal flow graph is shown in figure

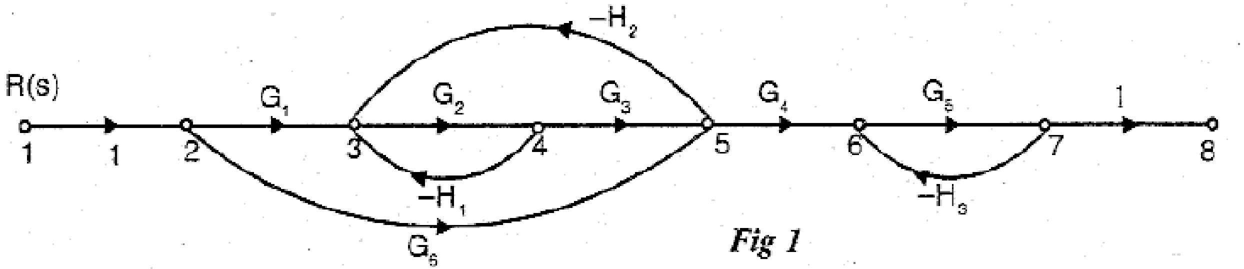


Fig 1

Forward Path Gains

Forward Path-1, (1-2-3-4-5-6-7-8) Gain of forward path-1, **P1 =G1G2G3G4G5**

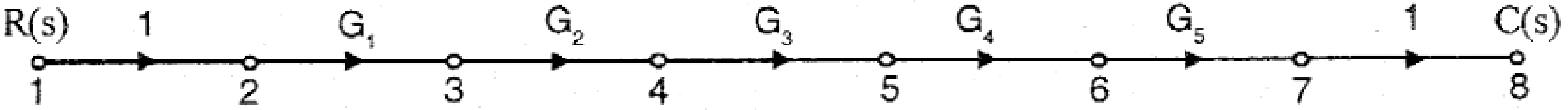


***Fig 2 :** Forward path-1.*

Forward Path-2, (1-2-5-6-7-8) Gain of forward path-2, **P2 = G4G5G6**

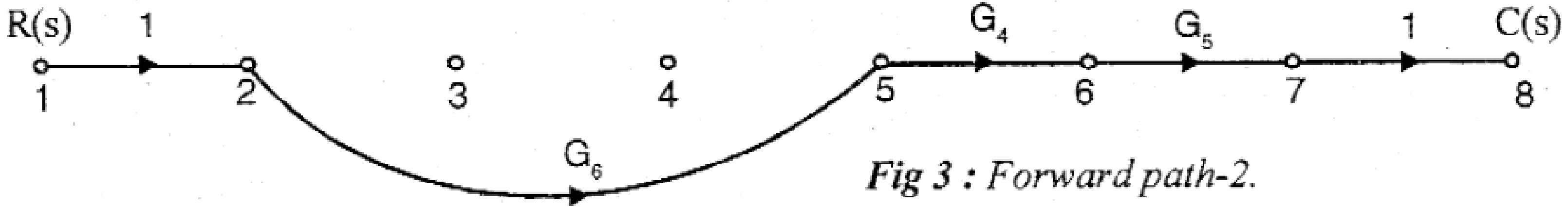


***Fig 3 :** Forward path-2.*

ATME College of Engineering

Individuals Loop

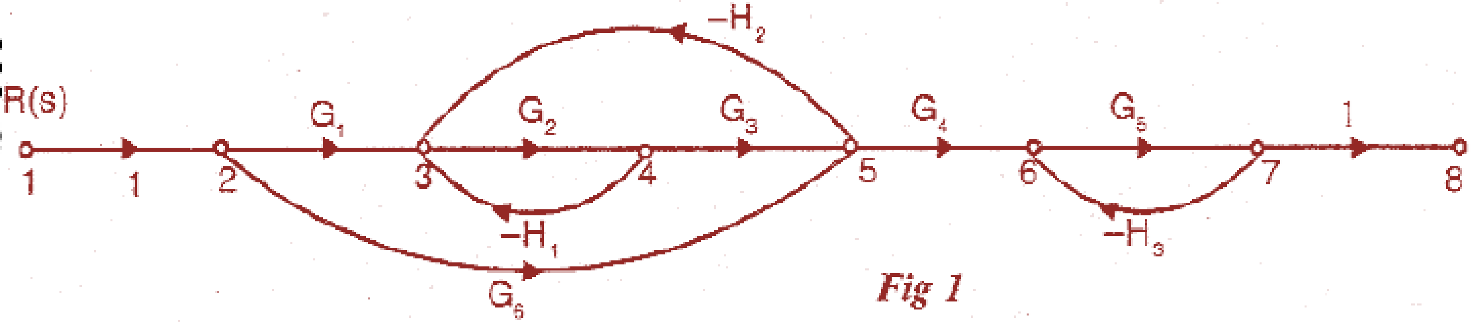


Fig 1

There are 3 Individuals Loops

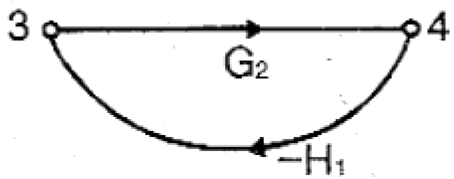


***Fig 4 :** Loop-1.*

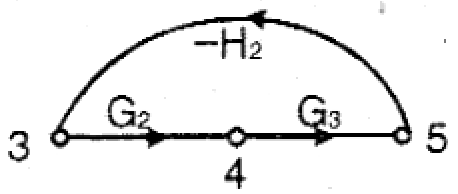


***Fig 5 :** Loop-2.*

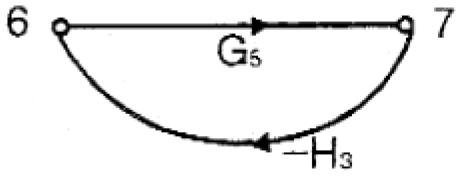


***Fig 6 :** Loop-3.*

Loop Gain of Individuals Loop-1, **L1 = -G2H1** (3-4-3)

Loop Gain of Individuals Loop-2, **L2 = -G2G3H2** (3-4-5-3)

Loop Gain of Individuals Loop-3, **L3 = -G5H3** (6-7-6)

Gain Products of Two Non-Individuals Loop

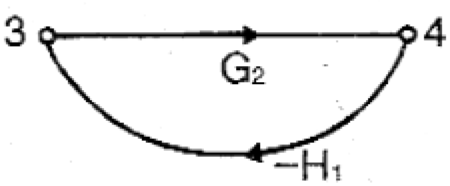


Fig 4 : Loop-1.

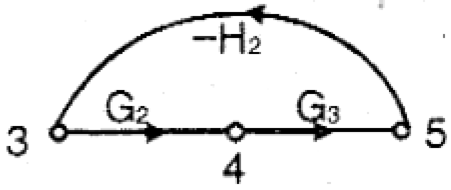


Fig 5 : Loop-2.

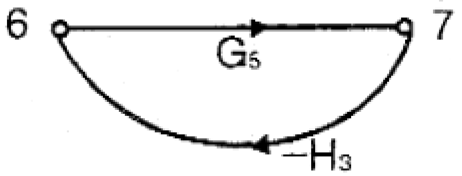


Fig 6 : Loop-3.

There are Two combinations of Two Non-touching loops

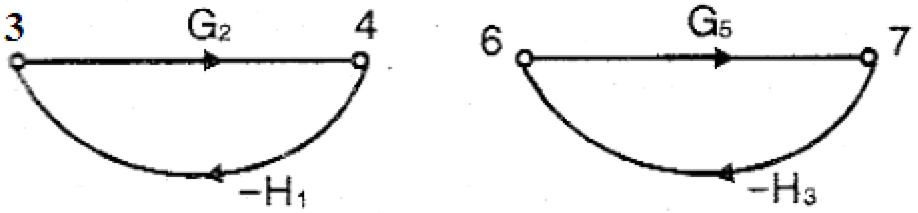


Fig.7 First combinations of 2 Non-touching loops

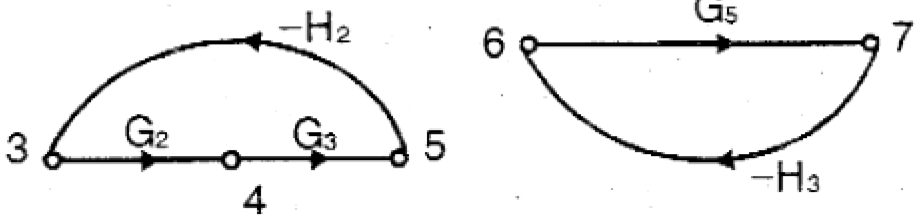


Fig.8 Second combinations of 2 Non-touching loops

Gain Product of 1st combinations of 2 Non-touching loops **L13 = L1*L3** =(-G2H1)*(-G5H3) = G2G5H1H3

Gain Product of 2nd combinations of 2 Non-touching loops **L23 = L2*L3** =(-G2G3H2)*(-G5H3) = G2G3G5H2H3

Mason's gain formula states the overall gain of the system (transfer function] as follows,

Overall gain, $$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

T = $T(s)$ = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

Determinant of SFG Δ = 1 – (Sum of individual loop gains)

$$+ \begin{pmatrix} \text{Sum of gain products of all possible} \\ \text{combinations of two non - touching loops} \end{pmatrix}$$

$$- \begin{pmatrix} \text{Sum of gain products of all possible} \\ \text{combinations of three non - touching loops} \end{pmatrix}$$

$+$ ....

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

Overall gain, $T = \frac{1}{\Delta} \sum_K P_K \Delta_K$ Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2)$$

Calculation of **Determinant of SFG** Δ and ΔK

$$\Delta = 1 - (L1+L2+L3) + (L13+L23)$$

$$= 1 - (-G2H1 - G2G3H2 - G5H3) + (G2G5H1H3 + G2G3G5H2H3)$$

$$= 1 + G2H1 + G2G3H2 + G5H3) + G2G5H1H3 + G2G3G5H2H3$$

Number of Forward path is 2 and so K =2

$\Delta 1 = 1$, Since **there is no part of graph which is not touching with 1st forward path**

Fig 2 : Forward path-1.

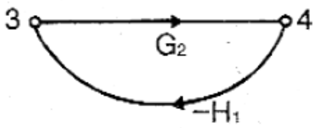


Fig 4 : Loop-1.

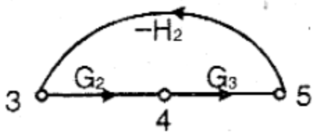


Fig 5 : Loop-2.

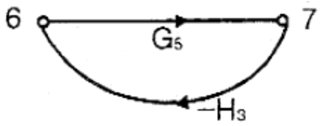


Fig 6 : Loop-3.

Calculation of **Determinant of SFG** Δ and ΔK

Number of Forward path is 2 and so K =2

$\Delta 2 = 1 - L1 = \mathbf{1-(-G2H1)}$

Since there is part of graph is which is not touching with 2nd forward path

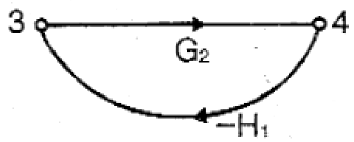


Fig 4 : Loop-1.

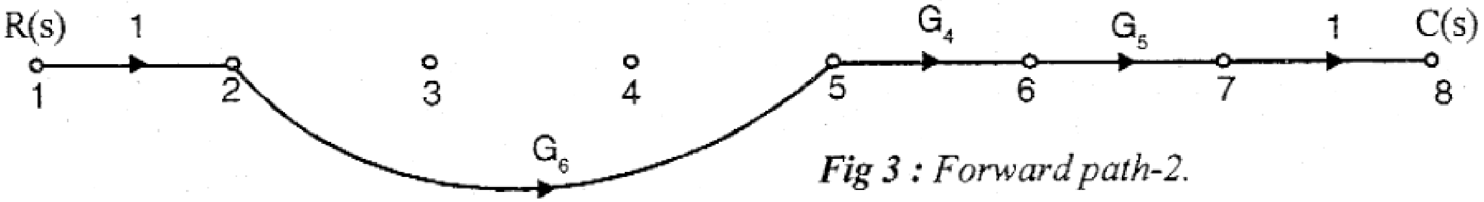


Fig 3 : Forward path-2.

Overall gain, $T = \frac{1}{\Delta}\sum_K P_K \Delta_K$

Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta}\sum_K P_K \Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_2G_4G_5\left[G_1G_3 + G_6/G_2 + G_6H_1\right]}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

2. Convert the block diagram to signal flow graph and determine the transfer function of the system using Mason's gain formula

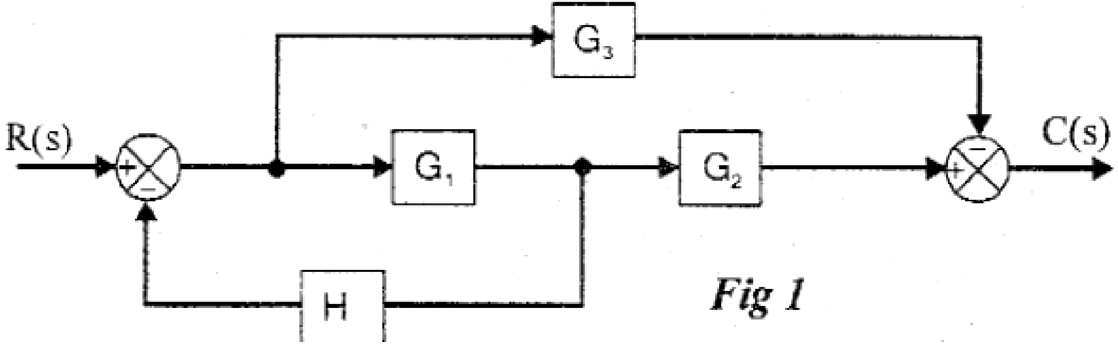


Fig 1

The nodes are assigned at input, output, at every point summing point & branch point as shown in Fig. 2.

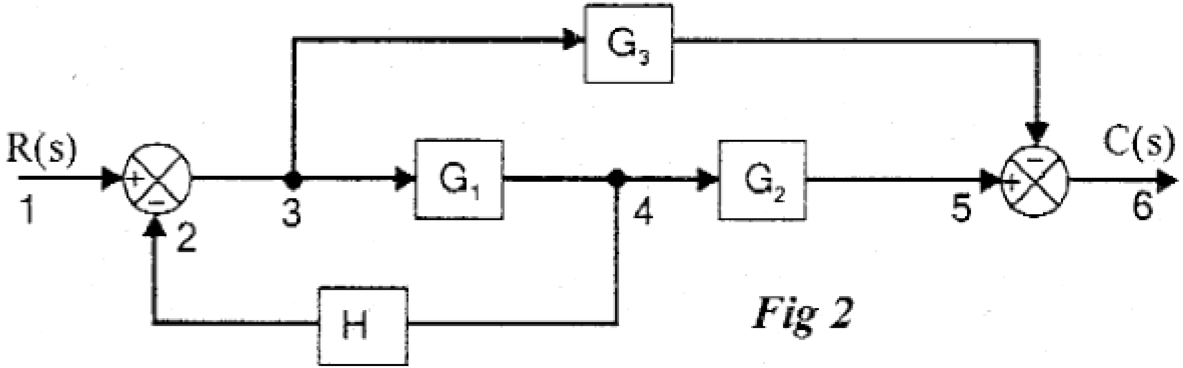


Fig 2

The nodes are assigned at input, output, at every point summing point & branch point as shown in Fig. 2.

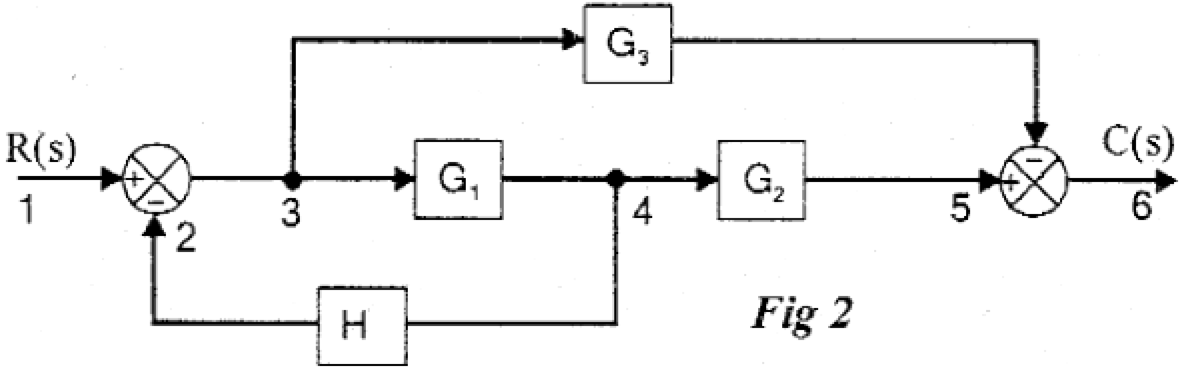


Fig 2

Signal flow graph is shown in figure 3

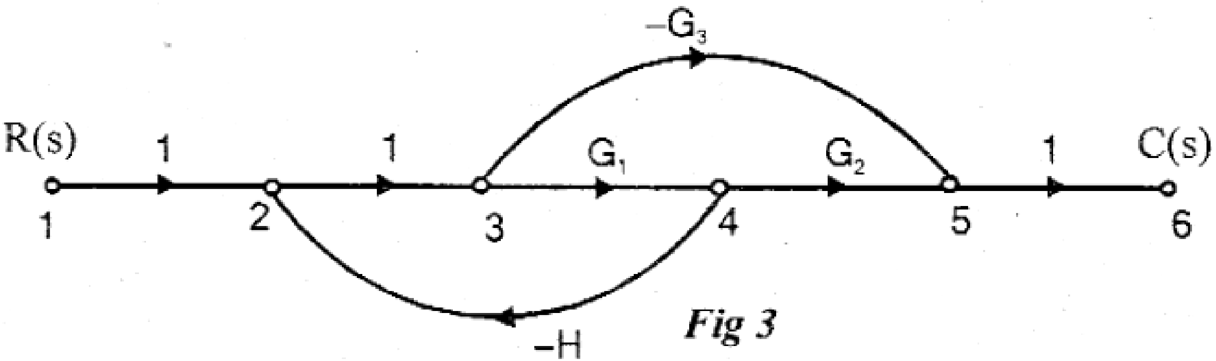


Fig 3

Signal flow graph is shown in figure 3

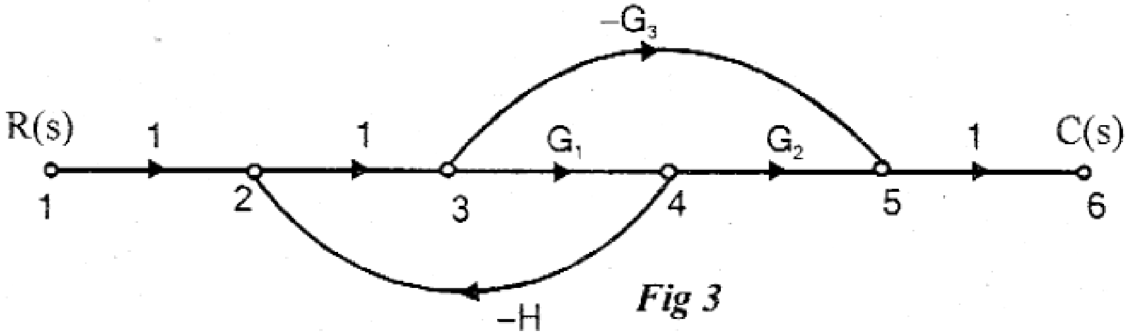


Fig 3

Forward Path Gains

Forward Path-1, Gain of forward path-1, **P1 = G1G2**

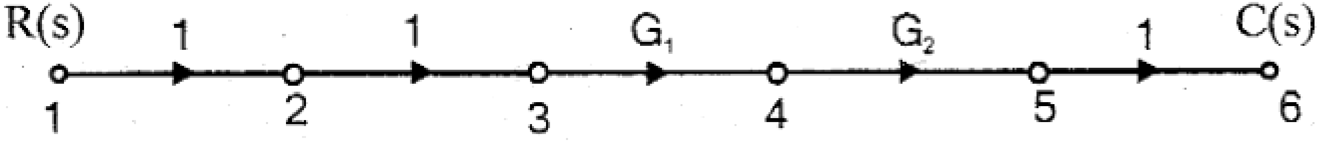
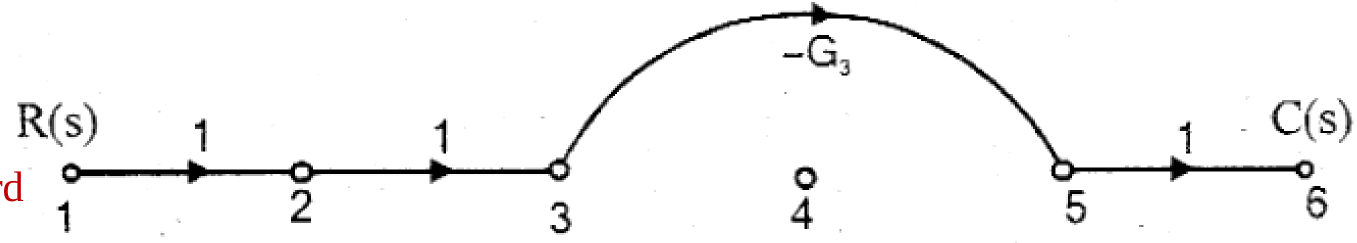


Fig 4 : *Forward path-1*



Forward Path-2, Gain of forward path-2, **P2 = -G3**

Fig 5 : *Forward path-2*

ATME College of Engineering

Department of EEE
Emitting Elite Energy

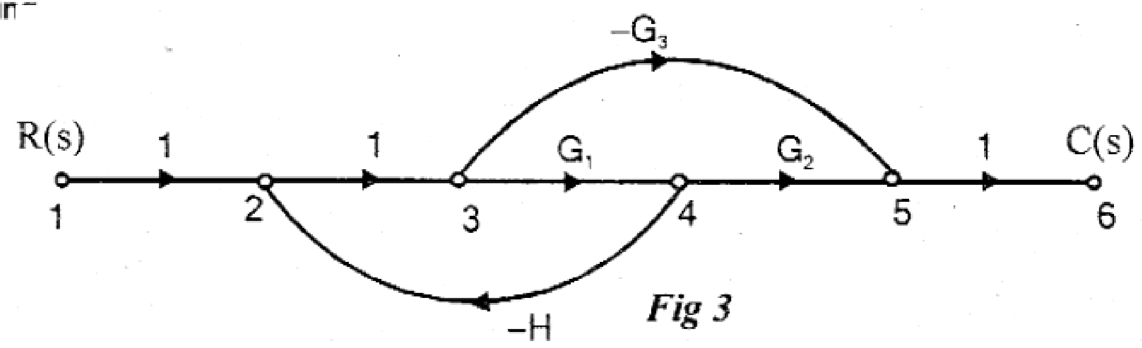


Fig 3

Individuals Loop

There is only one Individuals Loops

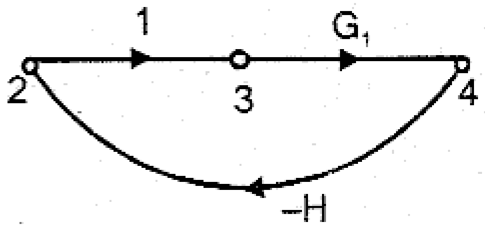


Fig 3 : loop-1

Loop Gain of Individuals Loop-1, **L1 = -G1*H**

Gain Products of Two Non-Individuals Loop

There are no combinations of non-touching loops

Mason's gain formula states the overall gain of the system (transfer function] as follows,

Overall gain, $T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$

T = $T(s)$ = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

Determinant of SFG Δ = 1 – (Sum of individual loop gains)

$+ \left(\begin{array}{l}\text{Sum of gain products of all possible} \\ \text{combinations of two non - touching loops}\end{array}\right)$

$- \left(\begin{array}{l}\text{Sum of gain products of all possible} \\ \text{combinations of three non - touching loops}\end{array}\right)$

$+ \ldots\ldots\ldots\ldots$

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

Overall gain, $T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$ Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2)$$

Calculation of **Determinant of SFG** Δ and ΔK

$$\Delta = 1 - (L1)$$

$$= 1 - (-G1H)$$

$$= 1 + G1H$$

Number of Forward path is 2 and so K =2

Δ1 = 1, Since there is no part of graph is which is not touching with 1st forward path

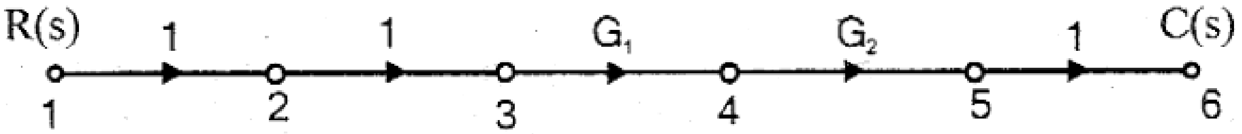


Fig 4 : Forward path-1

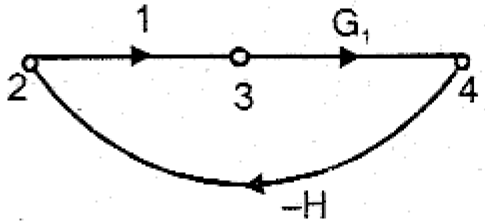


Fig 3 : loop-1

Δ2 = 1, Since there is no part of graph is which is not touching with 2nd forward path

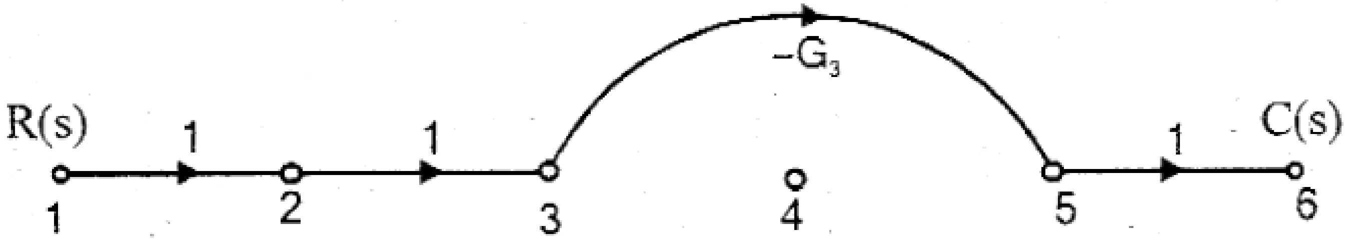


Fig 5 : Forward path-2

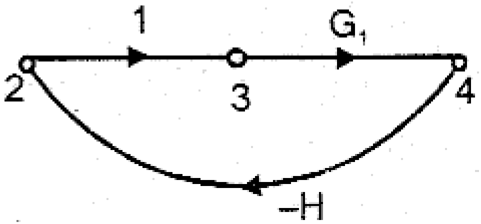


Fig 3 : loop-1

Overall gain, $T = \frac{1}{\Delta} \sum_K P_K \Delta_K$

Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta} (P_1\Delta_1 + P_2\Delta_2) = \frac{G_1G_2 - G_3}{1 + G_1H}$$

3. Convert the block diagram to signal flow graph and determine the transfer function of the system using Mason's gain formula

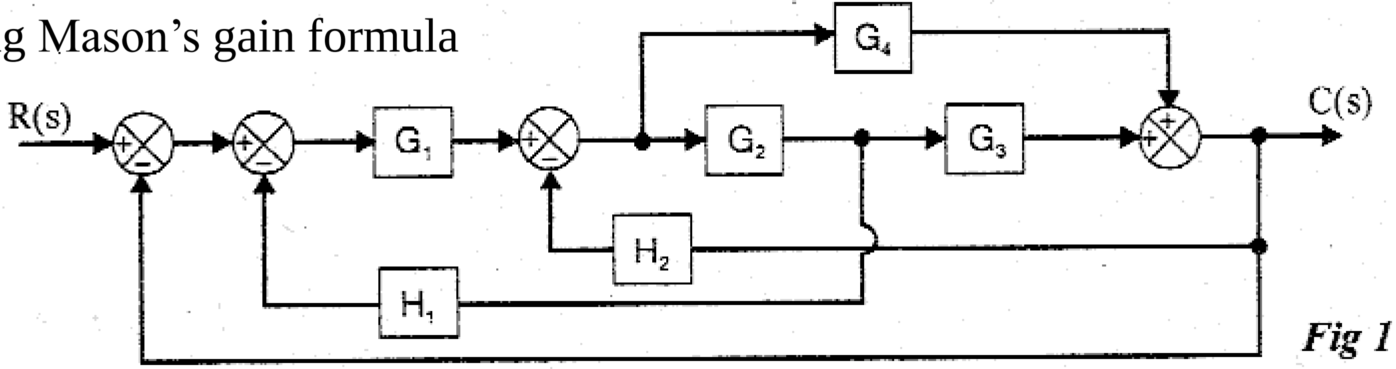


Fig 1

The nodes are assigned at input, output, at every point summing point & branch point as shown in Fig. 2.

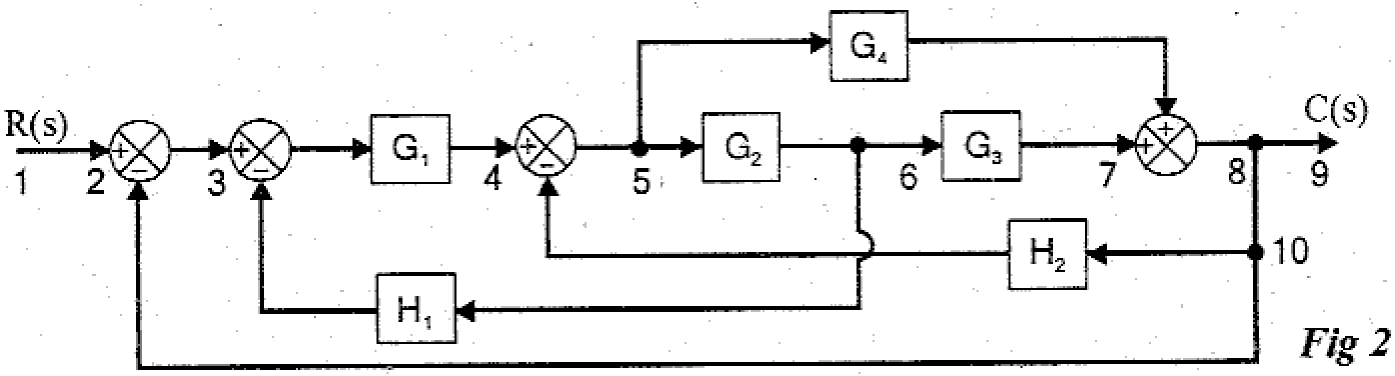


Fig 2

The nodes are assigned at input, output, at every point summing point & branch point as shown in Fig. 2.

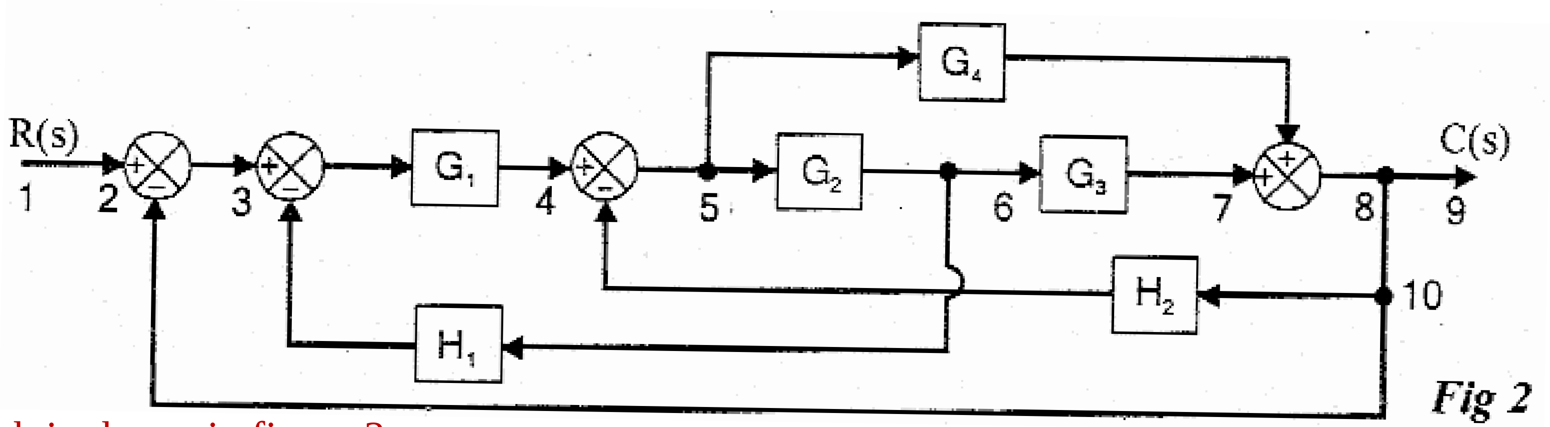


Fig 2

Signal flow graph is shown in figure 3

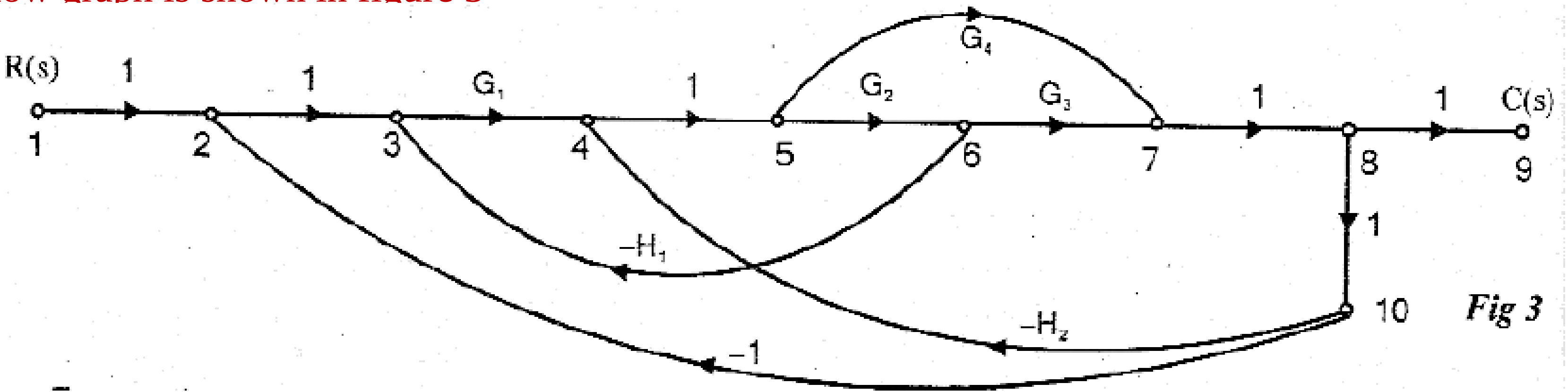


Fig 3

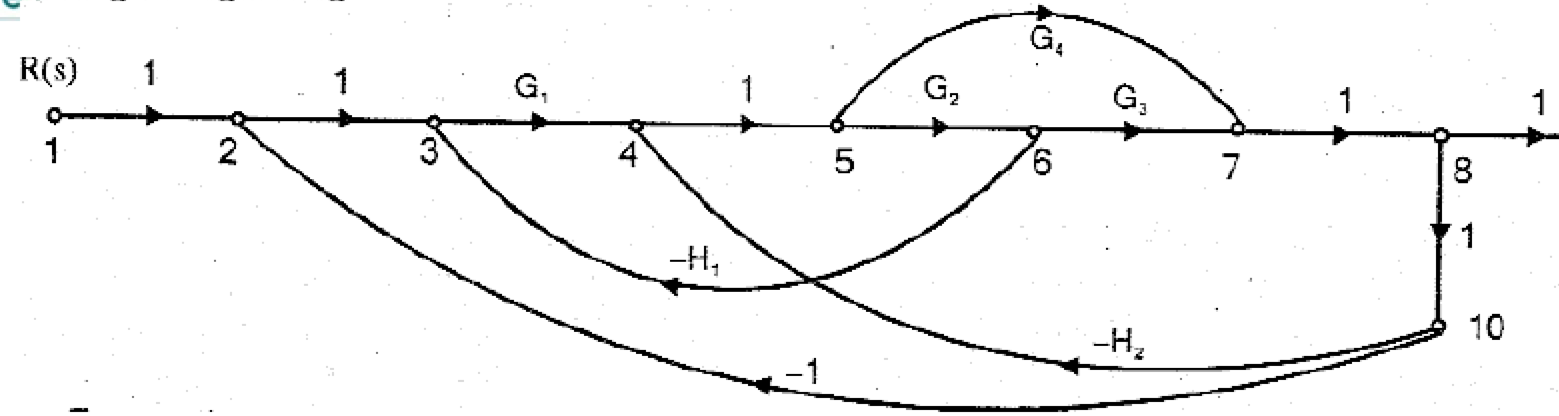
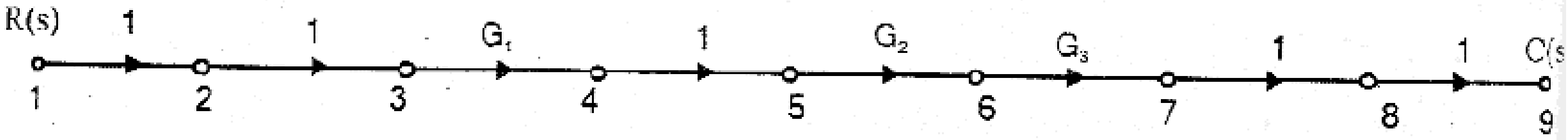
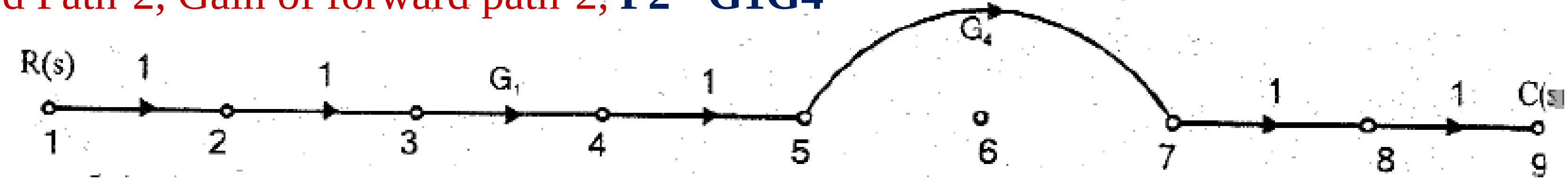


Fig 3

Forward Path-1, Gain of forward path-1, **P1 =G1G2G3**



Forward Path-2, Gain of forward path-2, **P2 =G1G4**



Individuals Loop Gain

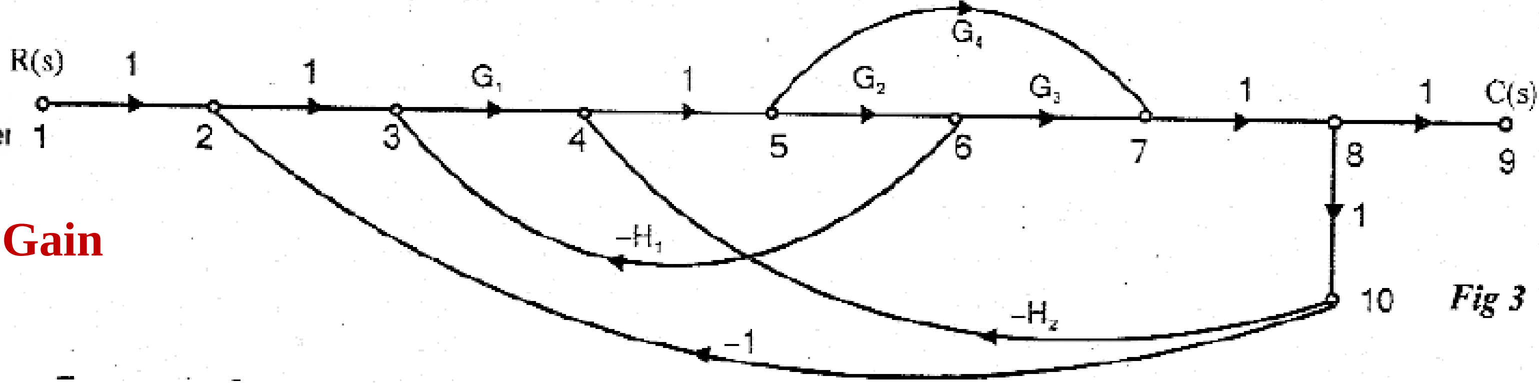


Fig 3

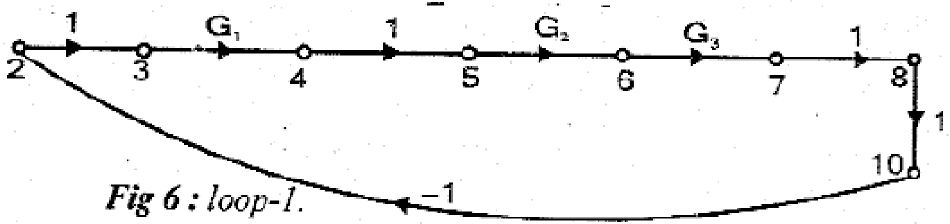


Fig 6 : loop-1.

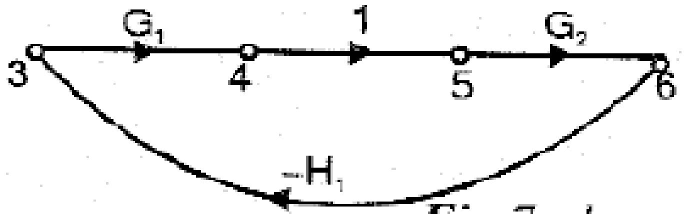


Fig 7 : loop-2.

Loop Gain of Individuals Loop-1, **L1 = -G1G2G3**

Loop Gain of Individuals Loop-2, **L2 = -G1G2H1**

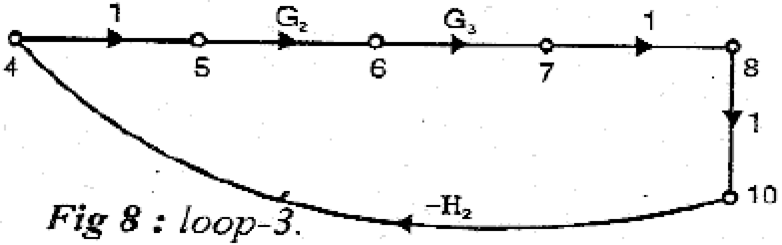


Fig 8 : loop-3.

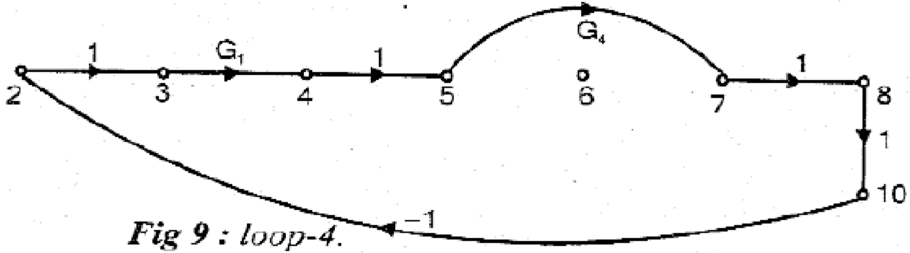


Fig 9 : loop-4.

Loop Gain of Individuals Loop-3, **L3 = -G2G3H2**

Loop Gain of Individuals Loop-4, **L4 = -G1G4**

ATME College of Engineering

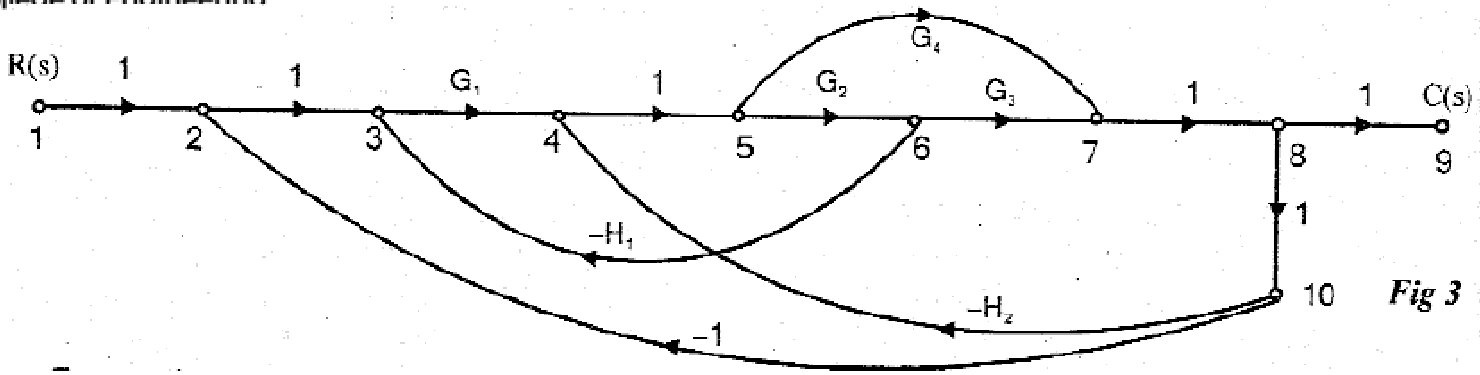


Fig 3

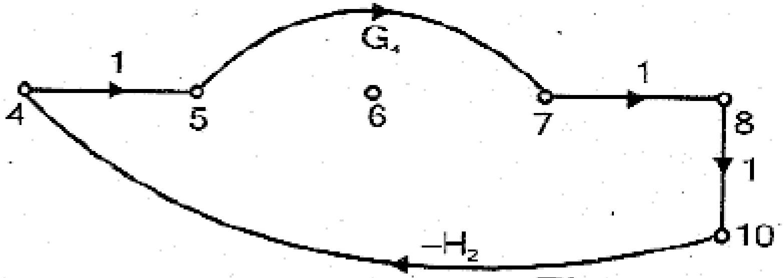


Fig 10 : loop-5.

Loop Gain of Individuals Loop-5, **L5 = -G4H2**

Gain Products of Two Non-Individuals Loop

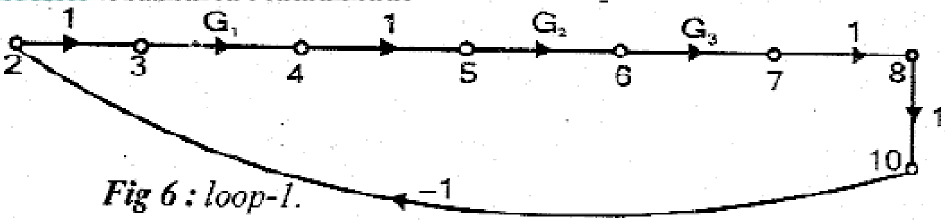


Fig 6 : loop-1.

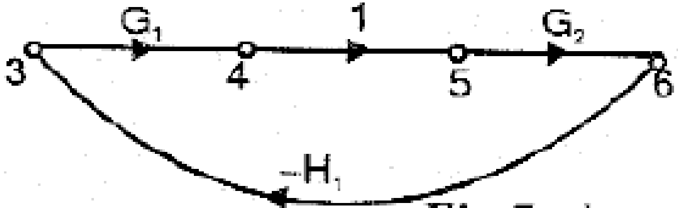


Fig 7 : loop-2.

Loop Gain of Individuals Loop-1, **L1 = -G1G2G3**

Loop Gain of Individuals Loop-2, **L2 = -G1G2H1**

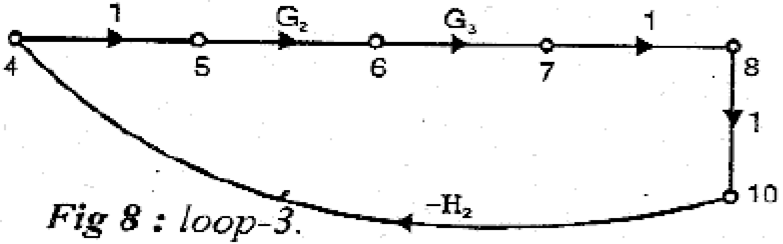


Fig 8 : loop-3.

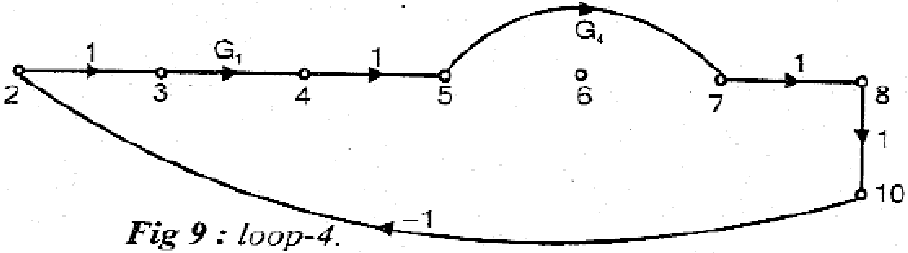


Fig 9 : loop-4.

Loop Gain of Individuals Loop-3, **L3 = -G2G3H3**

Loop Gain of Individuals Loop-4, **L4 = -G1G4**

Loop Gain of Individuals Loop-5, **L5 = -G4H2**

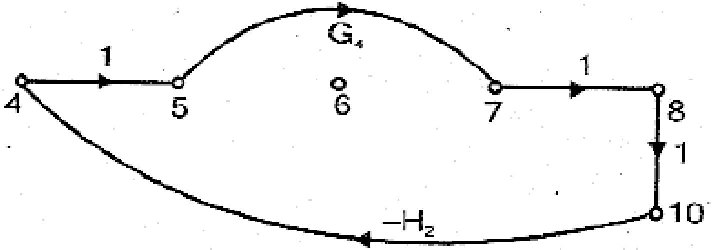


Fig 10 : loop-5.

There are no possible combinations of two Non Touching loops, three Non-touching loops

Mason's gain formula states the overall gain of the system (transfer function] as follows,

Overall gain, $T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$

T = $T(s)$ = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

Determinant of SFG Δ = 1 – (Sum of individual loop gains)

+ (Sum of gain products of all possible combinations of two non - touching loops)

– (Sum of gain products of all possible combinations of three non - touching loops)

+ ....

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

Overall gain, $T = \frac{1}{\Delta} \sum_K P_K \Delta_K$ Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta} (P_1\Delta_1 + P_2\Delta_2)$$

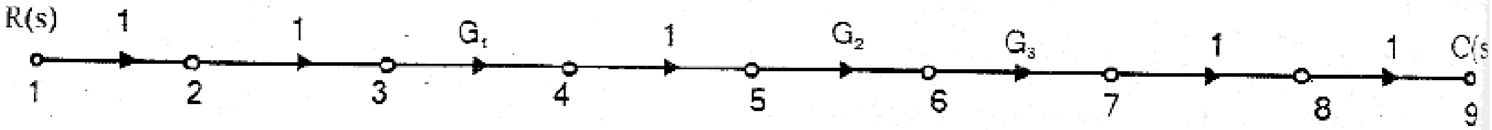
Calculation of **Determinant of SFG** Δ and ΔK

Δ = 1-[L1+L2+L3+L4+L5] = 1+G1G2G3+G1G2H1+G2G3H2+G1G4+G4H2

Δ1= Δ2 = 1, Since there is no part of graph which is not touching with forward path-1 & 2

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta} (P_1\Delta_1 + P_2\Delta_2) = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2}$$

Forward Path-1,



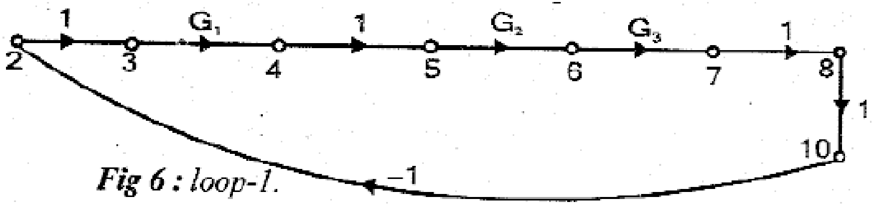


Fig 6 : loop-1.

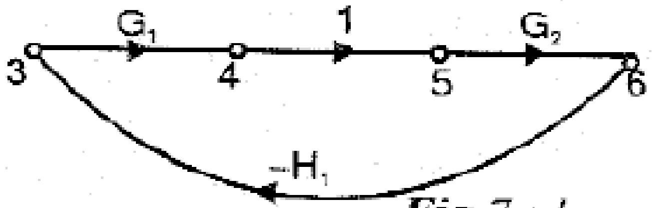


Fig 7 : loop-2.

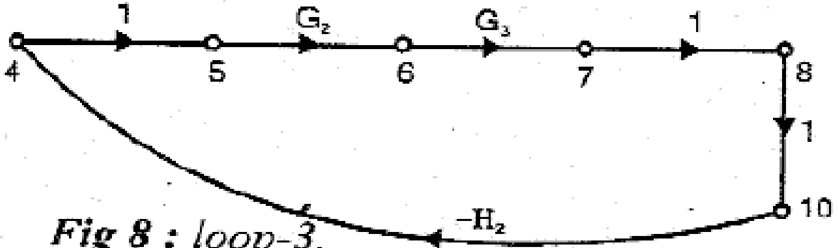


Fig 8 : loop-3.

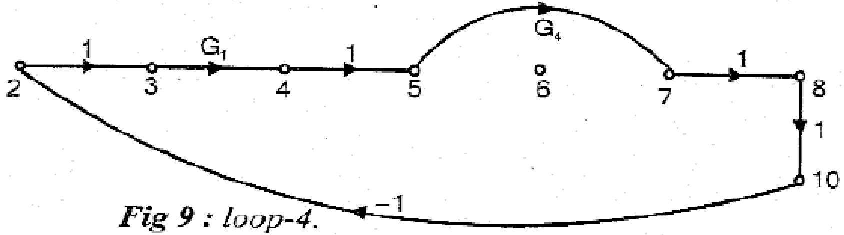


Fig 9 : loop-4.

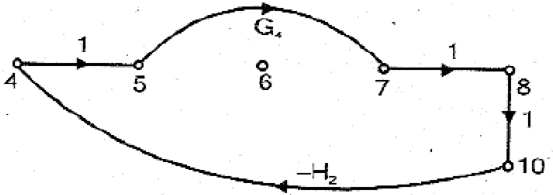
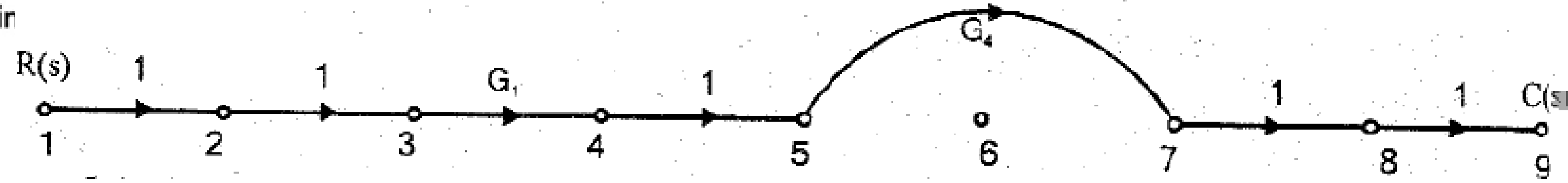


Fig 10 : loop-5.

$\Delta 1 = 1$, Since there is no part of graph is which is not touching with forward path-1

Forward Path-2,



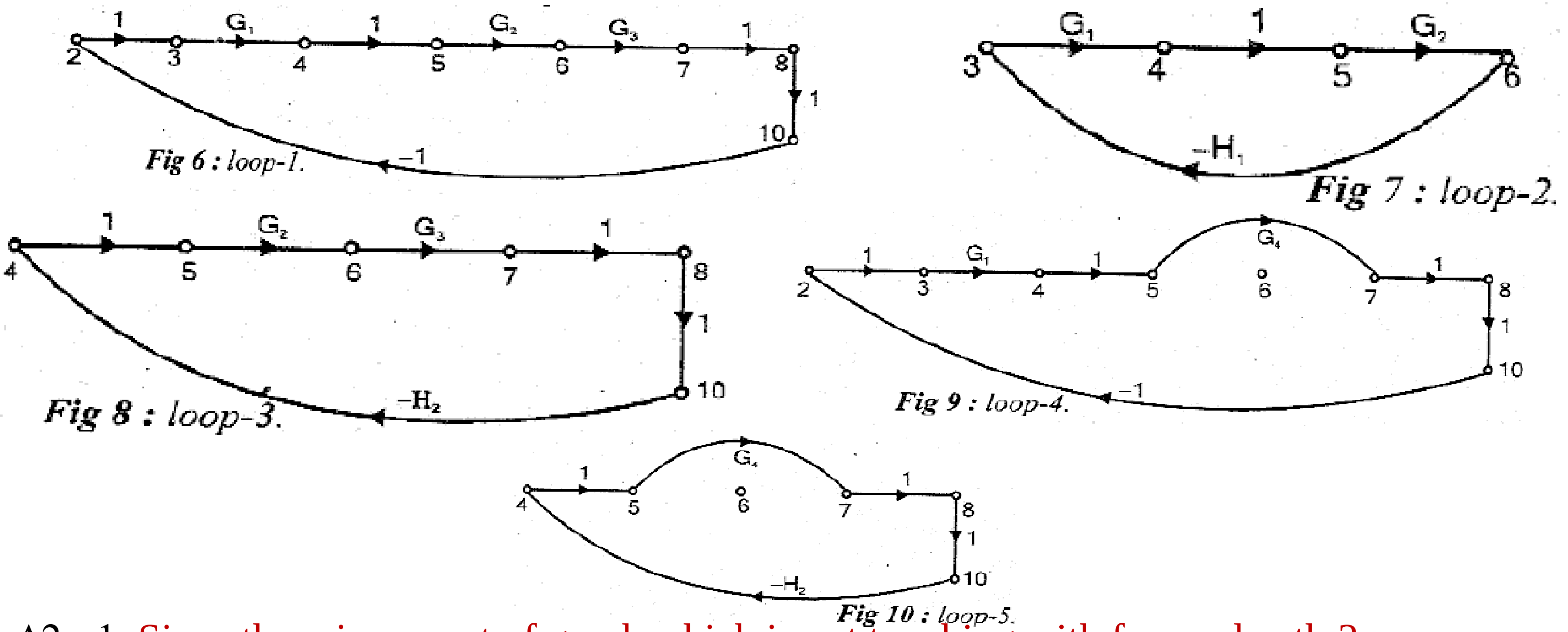


Fig 6 : loop-1.

Fig 7 : loop-2.

Fig 8 : loop-3.

Fig 9 : loop-4.

Fig 10 : loop-5.

Δ2= 1, Since there is no part of graph which is not touching with forward path-2

Overall gain, $T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$ Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

Calculation of **Determinant of SFG** Δ and ΔK

Δ = 1-[L1+L2+L3+L4+L5] = 1+G1G2G3+G1G2H1+G2G3H2+G1G4+G4H2

Δ1= Δ2 = 1, Since there is no part of graph is which is not touching with forward path-1 & 2

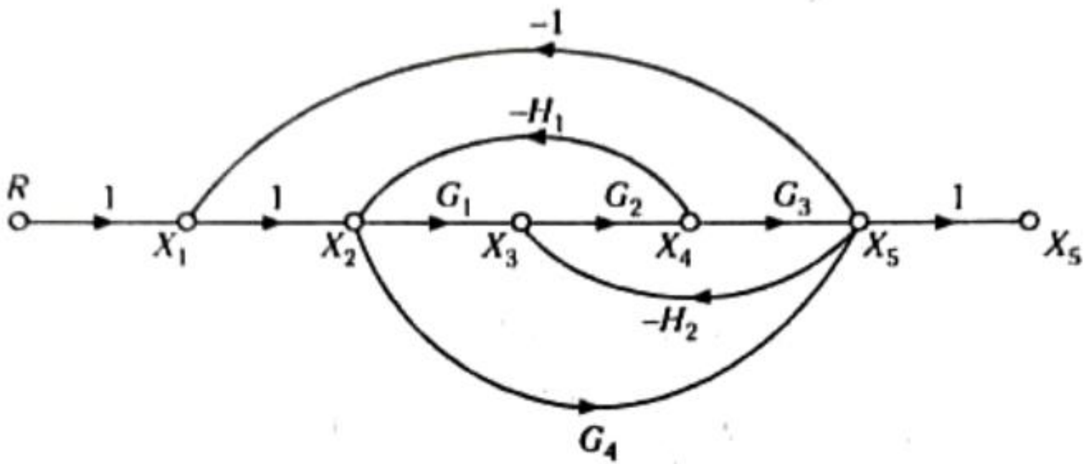
$$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2}$$

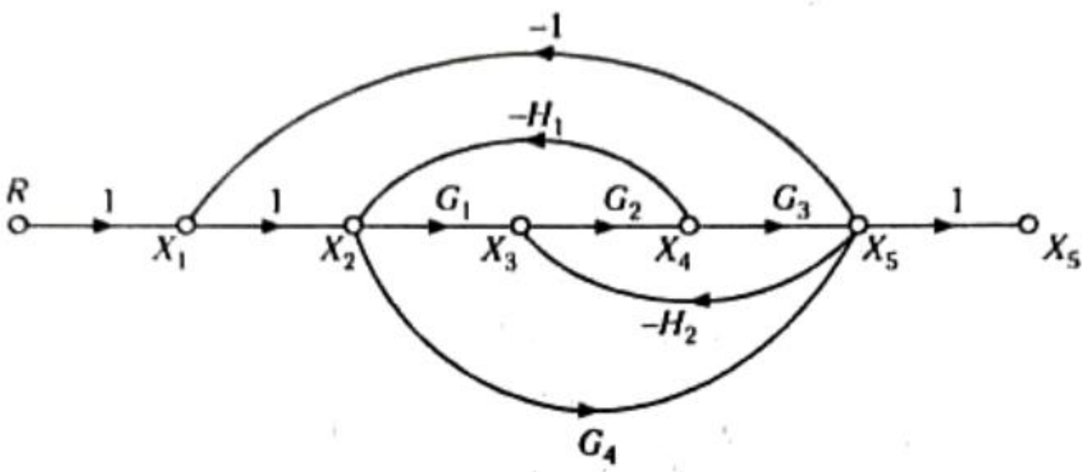
ATME College of Engineering

Department of EEE
Emitting Elite Energy

A system is described by following set of equations, where R is the input variable and X5 is the output variable. Draw the signal flow graph and determine the overall transfer function using Masons formula.

$X1 = R - X5$, $X2 = X1 - H1X4$, $X3 = G1X2 - H2X5$, $X4 = G2\ X3$, $X5 = G3\ X4 + G4\ X2$





Forward Paths

Forward Path-1, Gain of forward path-1, **P1 =G1G2G3**

Forward Path-2, Gain of forward path-2, **P2 = G4**

Loop Gain

L1 = -G1G2H1 L2 = -G2G3H2 L3 = -G4 L4 = G2G4H1H2 L5 = -G1G2G3

Mason's gain formula states the overall gain of the system (transfer function] as follows,

Overall gain, $$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$$

T = $T(s)$ = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

Determinant of SFG Δ = 1 – (Sum of individual loop gains)

+ (Sum of gain products of all possible combinations of two non - touching loops)

– (Sum of gain products of all possible combinations of three non - touching loops)

+ ....

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

Overall gain, $T = \frac{1}{\Delta} \sum_K P_K \Delta_K$

Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2)$$

Overall gain, $T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$ Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2)$$

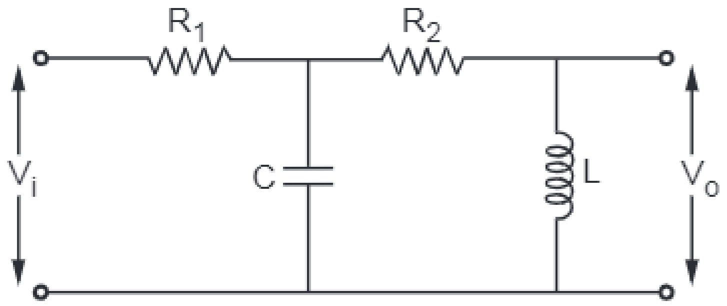
Calculation of **Determinant of SFG** Δ and ΔK

Δ = 1-[L1+L2+L3+L4+L5] = 1 + G1G2H1 + G2G3H2 + G4 - G2G4H1H2 + G1G2G3

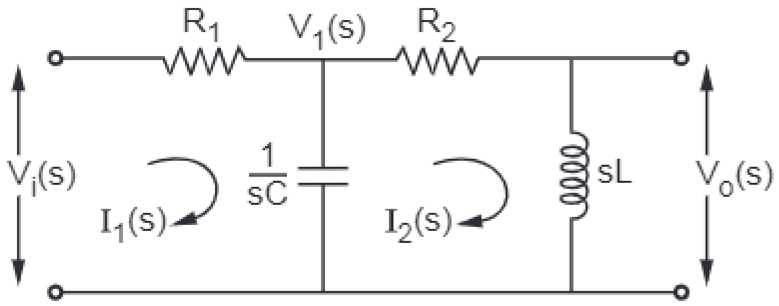
Δ1= Δ2 = 1, Since there is no part of graph is which is not touching with forward path-1 & 2

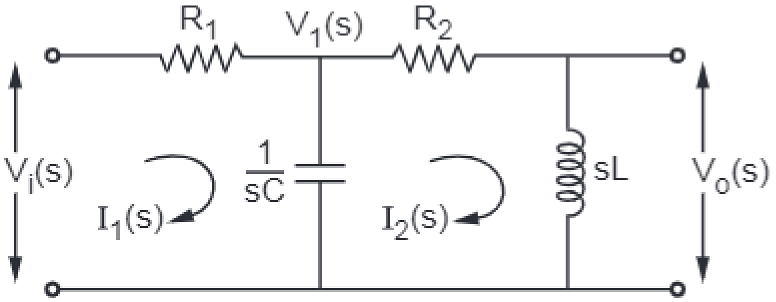
$$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2) = \frac{G_1 G_2 G_3 + G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 - G_2 G_4 H_1 H_2 + G1G2G3}$$

Determine the overall transfer function for the given Network using Masons formula.



Transformed Network in S-domain of the given network is as below(Laplace Transform)





Key Point *Assume the network variables alternately as the loop current and node voltage and then write the equations by analysing the horizontal and vertical branches alternately.*

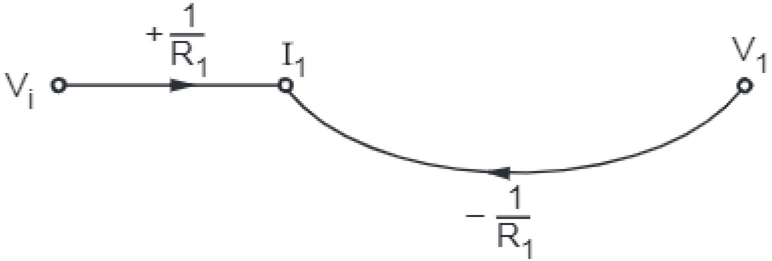
$$I_1(s) = \frac{(V_i - V_1)}{R_1} \quad \text{... (I)}$$

$$V_1 = (I_1 - I_2)\,\frac{1}{sC} \quad \text{... (II)}$$

$$I_2 = \frac{(V_1 - V_o)}{R_2} \quad \text{... (III)}$$

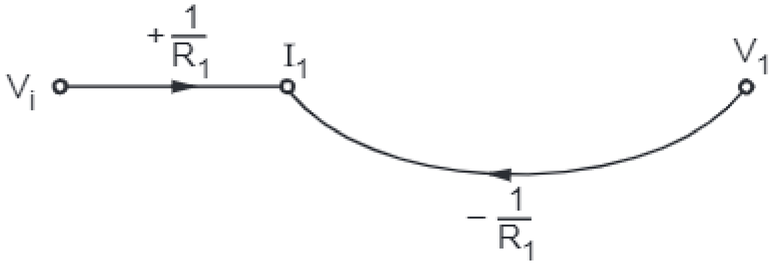
$$V_o = I_2\, sL \quad \text{... (IV)}$$

S.F.G. for equation (I)



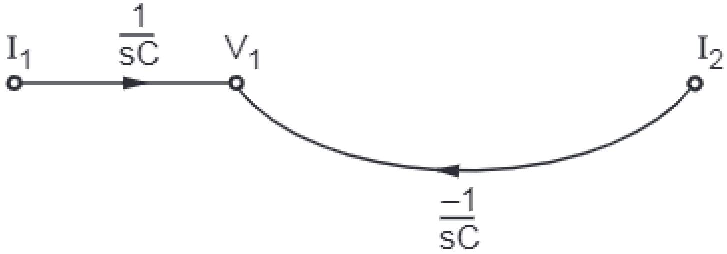
$$I_1(s) = \frac{(V_i - V_1)}{R_1} \quad \text{... (I)}$$

S.F.G. for equation (I)



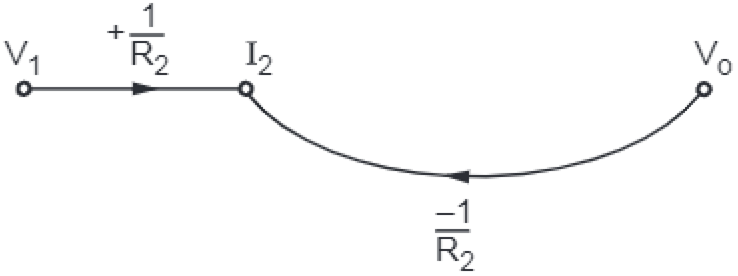
$$V_1 = (I_1 - I_2)\,\frac{1}{sC} \quad \text{... (II)}$$

S.F.G. for equation (II)



$$I_2 = \frac{(V_1 - V_o)}{R_2} \qquad \ldots \text{(III)}$$

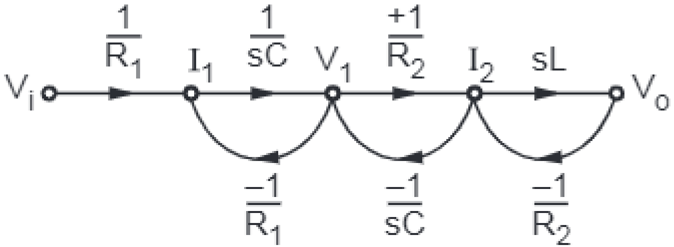
S.F.G. for equation (III)



$$V_o = I_2\, sL \qquad \ldots \text{(IV)}$$

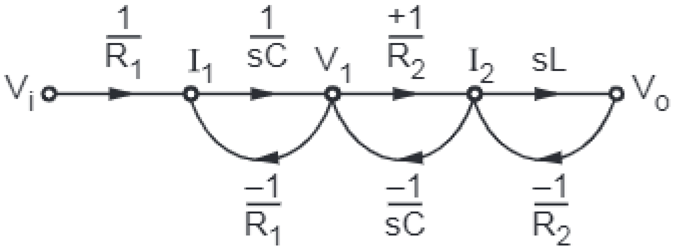
S.F.G. for equation (IV)

Use Mason's gain formula to find $\frac{V_o}{V_i}$.



$$\frac{V_o}{V_i} = \frac{\Sigma\ T_K\ \Delta_K}{\Delta};$$

$$\frac{V_o}{V_i} = \frac{\Sigma\ T_K\ \Delta_K}{\Delta}\ ; \qquad \text{Number of forward paths} = 1 \qquad \frac{V_o}{V_i} = \frac{T_1\ \Delta_1}{\Delta}$$



$$T_1 = \frac{L}{R_1\ R_2 C}$$

Individual feedback loops are,

$L_1 = -\dfrac{1}{sR_1\ C}$, $\quad L_2 = -\dfrac{1}{sR_2\ C}$, $\quad L_3 = -\dfrac{sL}{R_2}$ $\qquad$ L_1 and L_3 are non-touching.

ATME College of Engineering

Department of EEE
Emitting Elite Energy

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$$

$$\Delta = 1 + \frac{1}{sR_1 C} + \frac{1}{sR_2 C} + \frac{sL}{R_2} + \frac{L}{R_1 R_2 C}$$

As all loops are touching to T_1, $\Delta_1 = 1$,

$$\therefore \quad \frac{V_o}{V_i} = \frac{\dfrac{L}{R_1 R_2 C}}{1 + \dfrac{1}{sR_1 C} + \dfrac{1}{sR_2 C} + \dfrac{sL}{R_2} + \dfrac{L}{R_1 R_2 C}}$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{sL}{sR_1 R_2 C + R_2 + R_1 + s^2 L R_1 C + sL}$$

$$\therefore \quad \boxed{\frac{V_o(s)}{V_i(s)} = \frac{sL}{s^2LR_1C + s[L + R_1R_2C] + (R_1 + R_2)}}$$